ANNOUNCEMENTS

1. Final Exam - Dec 8 3:30-6:00 in Math 100.
   - Office Hours in Math Annex 1102
     Dec 4, 5, 6, 10:00-12:00
   - Exam Hardship, A&D.

   - PreCalculus Algebra: Do this Webwork.
     (not for Markus)
   - Q9 questions: 1, 3, 6, 9, 12

Today

• Quiz
• Linear Approximations
LINEAR APPROXIMATIONS

SMALL-ANGLE APPROXIMATION

\[ \sin(\theta) \approx \theta \text{ for tiny/small values of } \theta. \]

approximate a line

a function

(c.f. pendulum in dynamics).

BTW, Have you seen the Foucault Pendulum in the Physics Building?
SMALL-ANGLE APPROXIMATION

1. SKETCH $y = \sin \Theta$ and $y = \Theta$.

For small $\Theta$, $\sin \Theta \approx \Theta$. 
SMALL-ANGLE APPROXIMATION

Why is this true?

1. \[ \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \] (HW 4...)

say:

for small \( \theta \):

\[ \frac{\sin \theta}{\theta} \approx 1 \]

\[ \sin \theta \approx \theta \]

\[ \to \text{Non-rigorous argument based on limit approximation.} \]
2. Look at the derivative of \( y = \sin \theta \) at \( \theta = 0 \).

\[ y' = \cos \theta \implies y'(0) = \cos 0 = 1. \]

Note that if \( y = \theta \), \( y' = 1 \).

- \( \sin \theta \) and \( \theta \) are both 0 when \( \theta = 0 \).
- The derivatives of \( \sin \theta \) and \( \theta \) are both 1 when \( \theta = 0 \).
LINEAR APPROXIMATION

- If $f(x) = \sin x$, then the linear approximation of $f(x)$ near at $x = 0$ is

  \[ y - f(0) = f'(0)(x - 0) \]
  \[ y - \sin(0) = \cos(0)(x - 0) \]
  \[ y = 0 = 1(x - 0) \]
  \[ y = x. \]

DEFINITION The linear approximation of $f(x)$ at $x = a$ is

\[ f(x) \approx f(a) + f'(a)(x - a) \]

- tangent to the function at $x = a$. 
**LINEAR APPROX**

\[ L(x) = f(a) + f'(a)(x-a) \]

- \[ L(a) = f(a) + f'(a)(a-a) \]
  \[ = f(a) + f'(a) \cdot 0 \]
  \[ = f(a) \]  \(\text{Linear approx matches the y-value at } x=a\)

- \[ L'(x) = 0 + f'(a) \]
  \[ \Rightarrow L'(a) = f'(a) \]  \(\text{Linear approx matches the derivative of } f(x) \text{ at } x=a\)
Check-in

Suppose \( f(x) \) is a function, and its linear approximation near \( x = 5 \) is

\[ L(x) = 3x - 9. \]

(a) What is \( f(5) \)?

\[ f(5) = L(5) = 3 \cdot 5 - 9 = 15 - 9 = 6. \]

(b) What is \( f'(5) \)?

\[ f'(5) = L'(5) = 3 \]

(c) What is \( f(4) \)?

\[ f(4) \approx L(4) = 3 \cdot 4 - 9 = 12 - 9 = 3. \]
- Linear approx is valid for approximations only "close" to $x = a$.

- Error in our approximations & Taylor Remainder Theorem.

Ex. Estimate $\sqrt{4.1}$.

$f(x) = \sqrt{x}$. $f'(x) = \frac{1}{2\sqrt{x}}$

$L(x) = f(4) + f'(4)(x - 4)$

$= 2 + \frac{1}{4} (x - 4)$
\[ f(4.1) = \sqrt{4.1} \]

\[ \approx L \left( 4.1 \right) = 2 + \frac{1}{4} (4.1 - 4) \]

\[ = 2 + \frac{1}{4} (0.1) \]

\[ = 2 + \frac{1}{4} \cdot \frac{1}{10} = 2 + \frac{1}{40} \]

\[ \sqrt{4.1} = 2.0248456731 \ldots \text{ computer } = 2.025 \]

\[ \approx \text{ Close!} \]

\[ \text{Our approximation is an over-estimate} \]
OVER/UNDER-ESTIMATE?

- Concavity!

C. U.

C. D.

overestimate

underestimate

\[ \alpha \]
Reflection

Write a two sentence summary of what the key points of today's class are.