The remainder of this page has been left blank for your workings.
Very short answer questions

1. [2 marks] Each part is worth 1 marks. Please write your answers in the boxes.

(a) Let \( y = \cos(\log(\sin(x))) \). Compute \( \frac{dy}{dx} \).

(Recall: \( \log x = \log_e x = \ln x \).)

Answer: \(-\sin(\log(\sin(x))) \cdot \frac{\cos x}{\sin x}\)

Solution:

\[
\frac{dy}{dx} = -\sin(\log(\sin(x))) \cdot \frac{d}{dx}(\log(\sin x)) \\
= -\sin(\log(\sin(x))) \cdot \left(\frac{\cos x}{\sin x}\right)
\]

(b) If \( \sin(x) + y^2 = e^{x-y+1} \) compute \( \frac{dy}{dx} \) at \((x, y) = (0, 1)\).

Answer: \( \frac{dy}{dx} = 0 \)

Solution: Differentiate the equation

\[
\cos(x) + 2y \frac{dy}{dx} = e^{x-y+1} \cdot \left(1 - \frac{dy}{dx}\right)
\]

So when \( x = 0, y = 1 \):

\[
1 + 2\frac{dy}{dx} = e^0 \cdot \left(1 - \frac{dy}{dx}\right) \\
0 = \frac{dy}{dx}
\]
Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.
   (a) Find \( f'(x) \) if \( f(x) = (x + 1)^{\tan(x)} \).

**Solution:** We use logarithmic differentiation; so

\[
\log f(x) = \log(x + 1) \cdot \tan x
\]

Then differentiate to obtain

\[
\frac{f'(x)}{f(x)} = \frac{d}{dx} \left[ \log(x + 1) \cdot \tan x \right] = (\sec x)^2 \cdot \log(x + 1) + \frac{\tan x}{x + 1}
\]

In conclusion:

\[
f'(x) = f(x) \cdot \left( (\sec x)^2 \cdot \log(x + 1) + \frac{\tan x}{x + 1} \right)
\]

(b) For what values of \( x \) does the derivative of \( \arctan(\sqrt{x + 1}) \) exist?

**Solution:** This is just the chain rule:

\[
\frac{d}{dx} \arctan(\sqrt{x + 1}) = \frac{1}{1 + (\sqrt{x + 1})^2} \cdot \frac{1}{2\sqrt{x + 1}}
\]

So derivative exists provided \( x > -1 \). Hence the derivative exists on \((-1, \infty)\).
Long answer question — you must show your work

3. [4 marks] Let \( g(x) = \frac{x + 2}{2x + 3} \). Compute \( g'(x) \) using the definition of the derivative. No marks will be given for use of derivative rules, but you may use them to check your answer.

Solution:

\[
g'(x) = \lim_{h \to 0} \frac{g(x + h) - g(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{x + h + 2}{2x + 2h + 3} - \frac{x + 2}{2x + 3} \right]
\]

\[
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{(x + h + 2)(2x + 3) - (x + 2)(2x + 2h + 3)}{(2x + 2h + 3)(2x + 3)} \right]
\]

\[
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{(2x^2 + 3x + 2xh + 3h + 4x + 6) - (2x^2 + 2xh + 3x + 4x + 4h + 6)}{(2x + 2h + 3)(2x + 3)} \right]
\]

\[
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{-h}{(2x + 2h + 3)(2x + 3)} \right]
\]

\[
= \lim_{h \to 0} \left[ \frac{-1}{(2x + 2h + 3)(2x + 3)} \right]
\]

\[
= \frac{-1}{(2x + 3)^2}
\]
First Name: _______________________ Last Name: _______________________

Student-No: ______________________ Section: _________________________

Grade: __________________________

The remainder of this page has been left blank for your workings.
Very short answer questions

1. **2 marks** Each part is worth 1 marks. Please write your answers in the boxes.
   
   (a) Let \( y = \sqrt{\cos(\log(x))} \). Compute \( \frac{dy}{dx} \).
   
   (Recall: \( \log x = \log_e x = \ln x \).)

   **Answer:** \( -\frac{\sin(\log(x))}{2x\sqrt{\cos(\log(x))}} \)

   **Solution:**
   
   \[
   \frac{dy}{dx} = \frac{1}{2\sqrt{\cos(\log(x))}} \frac{d}{dx} (\cos(\log(x)))
   \]
   
   \[
   = \frac{-\sin(\log(x))/x}{2\sqrt{\cos(\log(x))}}
   \]

   (b) If \( x^2 - 2y = e^{xy} \) compute \( \frac{dy}{dx} \) at \( (x, y) = (-1, 0) \).

   **Answer:** \( \frac{dy}{dx} = -2 \)

   **Solution:** Differentiate the equation
   
   \[
   2x - 2y' = e^{xy}(y + xy')
   \]

   Plug in \( x = -1 \) and \( y = 0 \)

   \[
   2(-1) - 2y' = e^0(0 - y')
   \]
   
   \[
   -2 - 2y' = -y'
   \]
   
   \[
   -2 = y'
   \]
Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

(a) Find $f'(x)$ if $f(x) = (x + 1)^{\log(x)}$
(Recall: $\log x = \log_e x = \ln x$.)

**Solution:** We use logarithmic differentiation; so

$$\log f(x) = \log(x) \cdot \log(x + 1)$$

Then differentiate to obtain

$$\frac{f'(x)}{f(x)} = \frac{d}{dx} \left[ \log(x) \cdot \log(x + 1) \right]$$

$$= \frac{1}{x} \cdot \log(x + 1) + \log(x) \frac{1}{x + 1}$$

In conclusion:

$$f'(x) = f(x) \cdot \left( \frac{\log(x + 1)}{x} + \frac{\log(x)}{x + 1} \right)$$

$$= (x + 1)^{\log(x)} \cdot \left( \frac{\log(x + 1)}{x} + \frac{\log(x)}{x + 1} \right)$$

(b) For what values of $x$ does the derivative of $\arctan(\sqrt{x})$ exist?

**Solution:** Note that $\arctan(\sqrt{x})$ is only defined when $x \geq 0$. Apply the chain rule:

$$\frac{d}{dx} \arctan(\sqrt{x}) = \frac{1}{1 + (\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{1 + x} \cdot \frac{1}{2\sqrt{x}}$$

So derivative exists provided $x > 0$. Hence the derivative exists on $(0, \infty)$. 

Long answer question — you must show your work

3. 4 marks Let \( g(x) = \sqrt{1-x} \). Compute \( g'(x) \) using the definition of the derivative. No marks will be given for use of derivative rules, but you may use them to check your answer.

Solution:

\[
g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \\
= \lim_{h \to 0} \frac{\sqrt{1-(x+h)} - \sqrt{1-x}}{h} \\
= \lim_{h \to 0} \frac{\sqrt{1-(x+h)} - \sqrt{1-x}}{h} \cdot \frac{\sqrt{1-(x+h)} + \sqrt{1-x}}{\sqrt{1-(x+h)} + \sqrt{1-x}} \\
= \lim_{h \to 0} \frac{1}{h} \cdot \frac{1-(x+h) - (1-x)}{\sqrt{1-(x+h)} + \sqrt{1-x}} \\
= \lim_{h \to 0} \frac{1}{h} \cdot \frac{-h}{\sqrt{1-(x+h)} + \sqrt{1-x}} \\
= \lim_{h \to 0} \frac{-1}{\sqrt{1-(x+h)} + \sqrt{1-x}} \\
= \frac{-1}{2\sqrt{1-x}}
\]
Very short answer questions

1. 2 marks Each part is worth 1 marks. Please write your answers in the boxes.

(a) Let \( y = \sin(\log(\sin(x))) \). Compute \( \frac{dy}{dx} \).
(Recall: \( \log x = \log_e x = \ln x \).)

Answer: \( \cos(\log(\sin(x))) \cdot \frac{\cos x}{\sin x} \)

Solution:

\[
\frac{dy}{dx} = \cos(\log(\sin(x))) \cdot \frac{d}{dx}(\log(\sin(x)))
= \cos(\log(\sin(x))) \cdot \frac{\cos x}{\sin x}
\]

(b) If \( x + y + 1 = e^{(x^2 + y^2)} \) compute \( \frac{dy}{dx} \) at \( (x, y) = (0, 0) \).

Answer: \( \frac{dy}{dx} = -1 \)

Solution: Differentiate the equation with respect to \( x \):

\[
1 + \frac{dy}{dx} = (2x + 2y \frac{dy}{dx}) \cdot e^{x^2+y^2}.
\]

So, when \( x = 0 \) and \( y = 0 \):

\[
1 + \frac{dy}{dx} = (0 + 0 \cdot \frac{dy}{dx}) \cdot e^0
\]

Therefore,

\[
\frac{dy}{dx} = -1.
\]
Short answer questions — you must show your work

2. Each part is worth 2 marks.
   (a) Find \( f'(x) \) if \( f(x) = (\sin(x) + 2)x^2 \).

   **Solution:** We use logarithmic differentiation. First, take the logarithm
   \[
   \log f(x) = x^2 \cdot \log (\sin(x) + 2)
   \]
   Then differentiate to obtain
   \[
   \frac{f'(x)}{f(x)} = \frac{d}{dx} \left[ x^2 \cdot \log (\sin(x) + 2) \right] = 2x \cdot \log (\sin(x) + 2) + x^2 \cdot \frac{\cos(x)}{\sin(x) + 2}
   \]
   In conclusion:
   \[
   f'(x) = f(x) \cdot \left( 2x \cdot \log (\sin(x) + 2) + x^2 \cdot \frac{\cos(x)}{\sin(x) + 2} \right) \quad \text{or equivalently}
   \]
   \[
   = (\sin(x) + 2)x^2 \cdot \left( 2x \cdot \log (\sin(x) + 2) + x^2 \cdot \frac{\cos(x)}{\sin(x) + 2} \right)
   \]

   (b) For what values of \( x \) does the derivative of \( \arctan(\sqrt{x - 1}) \) exist?

   **Solution:** This is just the chain rule:
   \[
   \frac{d}{dx} \arctan(\sqrt{x - 1}) = \frac{1}{2\sqrt{x - 1}} \cdot \frac{1}{1 + (\sqrt{x - 1})^2}
   \]
   \[
   = \frac{1}{2x\sqrt{x - 1}}
   \]
   So derivative exists provided \( x \neq 0 \) and \( x > 1 \). Hence the derivative exists on \((1, +\infty)\).
3. 4 marks Let \( g(x) = \frac{2x}{x + 5} \). Compute \( g'(x) \) using the definition of the derivative. No marks will be given for use of derivative rules, but you may use them to check your answer.

Solution:

\[
g'(x) = \lim_{h \to 0} \frac{g(x + h) - g(x)}{h} \\
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{2(x + h)}{x + h + 5} - \frac{2x}{x + 5} \right] \\
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{2(x + h)(x + 5) - 2x(x + h + 5)}{(x + h + 5)(x + 5)} \right] \\
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{2x^2 + 2(h + 5)x + 10h - (2x^2 + 2x(h + 5))}{(x + h + 5)(x + 5)} \right] \\
= \lim_{h \to 0} \frac{10h}{(x + h + 5)(x + 5)} \\
= \frac{10}{(x + 5)^2}
\]
The remainder of this page has been left blank for your workings.
Very short answer questions

1. **2 marks** Each part is worth 1 marks. Please write your answers in the boxes.
   
   (a) Let \( y = \log(\cos(\log(x))) \). Compute \( \frac{dy}{dx} \).
   
   Answer: \( -\frac{\sin(\log(x))}{x \cos(\log(x))} \)
   
   **Solution:**
   \[
   \frac{dy}{dx} = \frac{1}{\cos \log x} \cdot \frac{d}{dx}(\cos(\log x))
   = -\frac{\sin(\log(x))/x}{\cos(\log(x))}
   \]

   (b) If \( x^2 + e^x - y = y^2 + x \) compute \( \frac{dy}{dx} \) at \((x, y) = (1, 1)\).

   Answer: \( \frac{dy}{dx} = 2/3 \)
   
   **Solution:** Differentiate the equation
   \[
   2x + e^x - y \left(1 - \frac{dy}{dx}\right) = 2y \frac{dy}{dx} + 1
   \]
   So when \( x = 1, y = 1 \):
   
   \[
   2 + e^0 \cdot \left(1 - \frac{dy}{dx}\right) = 2 \frac{dy}{dx} + 1
   \]
   
   \[
   3 - \frac{dy}{dx} = 2 \frac{dy}{dx} + 1
   \]
   
   \[
   \frac{dy}{dx} = 2/3.
   \]
2. \[4 \text{ marks} \] Each part is worth 2 marks.

(a) Find \( f'(x) \) if \( f(x) = (e^x + 1)\sin(x) \).

\[
\text{Solution: We use logarithmic differentiation; so}
\log f(x) = \log(e^x + 1) \cdot \sin x.
\]

Then differentiate to obtain

\[
\frac{f'(x)}{f(x)} = \frac{d}{dx} [\log(e^x + 1) \cdot \sin x] = \cos x \cdot \log(e^x + 1) + \frac{e^x \sin x}{e^x + 1}.
\]

In conclusion:

\[
f'(x) = f(x) \cdot \left( \cos x \cdot \log(e^x + 1) + \frac{e^x \sin x}{e^x + 1} \right) \quad \text{or equivalently}
\]

\[
= (e^x + 1)^\sin(x) \cdot \left( \cos x \cdot \log(e^x + 1) + \frac{e^x \sin x}{e^x + 1} \right).
\]

(b) For what values of \( x \) does the derivative of \( \arctan(\log x) \) exist?

(Recall: \( \log x = \log_e x = \ln x \).)

\[
\text{Solution: Just used the chain rule}
\]

\[
\frac{d}{dx} \arctan(\log x) = \frac{1}{1 + (\log x)^2} \cdot \frac{1}{x}
\]

So this blows up when \( x = 0 \). But the original function only exist for \( x > 0 \), so the derivative exists for all \( x > 0 \).
Long answer question — you must show your work

3. \(4\) marks Let \(g(x) = \frac{2x}{x + 3}\). Compute \(g'(x)\) using the definition of the derivative. No marks will be given for use of derivative rules, but you may use them to check your answer.

Solution:

\[
g'(x) = \lim_{h \to 0} \frac{g(x + h) - g(x)}{h} \\
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{2x + 2h}{x + h + 3} - \frac{x}{x + 3} \right] \\
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{(2x + 2h)(x + 3) - 2x(x + h + 3)}{(x + h + 3)(x + 3)} \right] \\
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{(2x^2 + 6x + 2xh + 6h) - (2x^2 + 2xh + 6x)}{(x + h + 3)(x + 3)} \right] \\
= \lim_{h \to 0} \frac{6h}{(x + h + 3)(x + 3)} \\
= \frac{6}{(x + 3)^2}
\]