The remainder of this page has been left blank for your workings.
Very short answer questions

1. 2 marks Each part is worth 1 marks. Please write your answers in the boxes.

   (a) Compute \( \lim_{x \to 1} \frac{2x}{\sqrt{3x^2 + 7}} \).

   Answer: \( \frac{2}{\sqrt{10}} = \frac{\sqrt{2}}{5} \)

   Solution:
   \[
   \lim_{x \to 1} \frac{2x}{\sqrt{3x^2 + 7}} = \lim_{x \to 1} \frac{2x}{\sqrt{3x^2 + 7}} = \frac{2}{\sqrt{\lim_{x \to 1}(3x^2 + 7)}} = \frac{2}{\sqrt{10}} = \frac{\sqrt{2}}{5}
   \]

   (b) Compute the limit \( \lim_{x \to -3} \frac{x^2 + 5x + 6}{x + 3} \).

   Answer: -1

   Solution: If try naively then we get 0/0, so we simplify first:
   \[
   \frac{x^2 + 5x + 6}{x + 3} = \frac{(x + 3)(x + 2)}{(x + 3)} = x + 2
   \]
   Hence the limit is \( \lim_{x \to -3} (x + 2) = -1 \).
Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

(a) Find the left-hand and right-hand limits of \( \frac{x}{\sqrt{x^2}} \) as \( x \to 0 \).

Answer: \( l = -1, r = 1 \)

Solution:

\[
\frac{x}{\sqrt{x^2}} = \begin{cases} 
\frac{x}{x} = 1 & x > 0 \\
\frac{x}{-x} = -1 & x < 0 
\end{cases}
\]

So left-hand limit is \(-1\) while right-hand limit is \(+1\).

(b) Evaluate \( \lim_{x \to -\infty} \frac{\sqrt{x^2 + 5} - x}{3x + 5} \)

Answer: \(-\frac{2}{3}\)

Solution: We divide by the highest power of the numerator, which is \( x \) and note that

\[
\frac{\sqrt{x^2 + 5}}{x} = -\sqrt{\frac{x^2 + 5}{x^2}} = -\sqrt{1 + \frac{5}{x^2}}.
\]

Since \( 1/x \to 0 \) and also \( 1/x^2 \to 0 \) as \( x \to -\infty \), we conclude that

\[
\lim_{x \to -\infty} \frac{\sqrt{x^2 + 5} - x}{3x + 5} = \lim_{x \to -\infty} \frac{-\sqrt{1 + \frac{5}{x^2}} - 1}{3 + \frac{5}{x}}
= \frac{-1 - 1}{3} = -\frac{2}{3}.
\]
Long answer question — you must show your work

3. 4 marks Compute the limit $\lim_{x \to -1} \frac{\sqrt{x^2 + 3} - 2}{2x + 2}$.

Answer: $-\frac{1}{4}$

**Solution:** If we try to do the limit naively we get $0/0$. Hence we must simplify.

\[
\frac{\sqrt{x^2 + 3} - 2}{2x + 2} = \frac{\sqrt{x^2 + 3} - 2}{2(x + 1)} \cdot \frac{\sqrt{x^2 + 3} + 2}{\sqrt{x^2 + 3} + 2} \\
= \frac{(x^2 + 3) - 4}{2(x + 1)(\sqrt{x^2 + 3} + 2)} \\
= \frac{(x^2 - 1)}{2(x + 1)(\sqrt{x^2 + 3} + 2)} \\
= \frac{x - 1}{2(\sqrt{x^2 + 3} + 2)}
\]

So the limit is

\[
\lim_{x \to -1} \frac{\sqrt{x - 2} - \sqrt{4 - x}}{x - 3} = \lim_{x \to -1} \frac{x - 1}{2(\sqrt{x^2 + 3} + 2)} \\
= \frac{-2}{2 \cdot 4} \\
= -\frac{1}{4}.
\]
First Name: __________________________ Last Name: __________________________
Student-No: ________________________ Section: __________________________

Grade:

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Very short answer questions

1. [2 marks] Each part is worth 1 marks. Please write your answers in the boxes.

(a) Compute \( \lim_{x \to 1} \sqrt{11x^2 + 5} \).

\[ \text{Answer: 4} \]

\textbf{Solution:} \\
\[ \lim_{t \to 1} \sqrt{11x^2 + 5} = \sqrt{\lim_{t \to 1} (11x^2 + 5)} \] \\
\[ = \sqrt{11 \lim_{t \to 1} (x^2)} + 5 \] \\
\[ = \sqrt{16} = 4. \]

(b) Compute the limit \( \lim_{x \to 4} \frac{x^2 - 16}{x - 4} \).

\[ \text{Answer: 8} \]

\textbf{Solution:} If try naively then we get 0/0, so we simplify first:

\[ \frac{x^2 - 16}{x - 4} = \frac{(x - 4)(x + 4)}{(x - 4)} = x + 4 \]

Hence the limit is \( \lim_{x \to 4} (x + 4) = 8. \)
Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.
   (a) Find the left-hand and right-hand limits of \( \frac{2x - 6}{|x - 3|} \) as \( x \to 3 \).

   \[
   \frac{2x - 6}{|x - 3|} = 2 \cdot \frac{x - 3}{|x - 3|} = \begin{cases} 
   2 \cdot \frac{x - 3}{3 - x} & \text{if } x < 3 \\
   2 \cdot \frac{x - 3}{x - 3} & \text{if } x > 3 
   \end{cases}
   \]

   So left-hand limit is \(-2\) while right-hand limit is \(+2\).

   \[
   \text{Answer: } l = -2, r = +2
   \]

   (b) Evaluate \( \lim_{x \to -\infty} \frac{5x + 4}{\sqrt{x^2 + 4} - x} \)

   \[
   \text{Solution: We divide by the highest power of the denominator, which is } x \text{ and note that} \\
   \frac{\sqrt{x^2 + 4}}{x} = -\sqrt{\frac{x^2 + 4}{x^2}} = -\sqrt{1 + \frac{4}{x^2}}.
   \]

   Since \( 1/x \to 0 \) and also \( 1/x^2 \to 0 \) as \( x \to -\infty \), we conclude that

   \[
   \lim_{x \to -\infty} \frac{5x + 4}{\sqrt{x^2 + 4} - x} = \lim_{x \to -\infty} \frac{5 + \frac{4}{x}}{-\sqrt{1 + \frac{4}{x^2}} - 1} = \frac{5}{-1 - 1} = -\frac{5}{2}.
   \]

   \[
   \text{Answer: } -5/2
   \]
Long answer question — you must show your work

3. [4 marks] Compute the limit \( \lim_{x \to 2} \frac{\sqrt{x - 1} - \sqrt{3 - x}}{x - 2} \).

\[ \text{Answer: 1} \]

\[ \text{Solution: If we try to do the limit naively we get 0/0. Hence we must simplify.} \]

\[ \frac{\sqrt{x - 1} - \sqrt{3 - x}}{x - 2} = \frac{\sqrt{x - 1} - \sqrt{3 - x}}{x - 2} \cdot \frac{\sqrt{x - 1} + \sqrt{3 - x}}{\sqrt{x - 1} + \sqrt{3 - x}} \]

\[ = \frac{(x - 1) - (3 - x)}{(x - 2)(\sqrt{x - 1} + \sqrt{3 - x})} \]

\[ = \frac{2x - 4}{(x - 2)(\sqrt{x - 1} + \sqrt{3 - x})} \]

\[ = \frac{2}{\sqrt{x - 1} + \sqrt{3 - x}} \]

So the limit is

\[ \lim_{x \to 2} \frac{\sqrt{x - 1} - \sqrt{3 - x}}{x - 2} = \lim_{x \to 2} \frac{2}{\sqrt{x - 1} + \sqrt{3 - x}} \]

\[ = \frac{2}{1 + 1} \]

\[ = 1. \]
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Grade:

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Very short answer questions

1.  

(a) Compute \( \lim_{x \to 1} \sqrt{2x + 6} \).

Answer: 2

Solution:

\[
\lim_{t \to 1} \sqrt{2t + 6} = \sqrt{\lim_{t \to 1} (2t + 6)} = \sqrt{2(1) + 6} = \sqrt{8} = 2.
\]

(b) Compute the limit \( \lim_{x \to -3} \frac{x^2 + 2x - 3}{x + 3} \).

Answer: -4

Solution: If try naively then we get 0/0, so we simplify first:

\[
\frac{x^2 + 2x - 3}{x + 3} = \frac{(x + 3)(x - 1)}{(x + 3)} = x - 1.
\]

Hence the limit is \( \lim_{x \to -3} (x - 1) = -4 \).
Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

(a) Find the left-hand and right-hand limits of \( \frac{|4x - 4|}{x - 1} \) as \( x \to 1 \).

\[
\begin{align*}
\text{Answer:} & \quad \lim_{x \to 1^-} \frac{|4x - 4|}{x - 1} = -4, \\
& \quad \lim_{x \to 1^+} \frac{|4x - 4|}{x - 1} = 4
\end{align*}
\]

Solution:

\[
\frac{|4x - 4|}{x - 1} = 4 \cdot \frac{|x - 1|}{x - 1} = \begin{cases} 
4 \cdot \frac{-(x - 1)}{x - 1} = -4 & x < 1 \\
4 \cdot \frac{x - 1}{x - 1} = +4 & x > 1
\end{cases}
\]

So left-hand limit is \(-4\) while right-hand limit is \(+4\).

(b) Evaluate \( \lim_{x \to -\infty} \frac{x}{\sqrt{4x^2 + 1} - 3x} \)

\[
\text{Answer:} \quad -\frac{1}{5}
\]

Solution: We divide by the highest power of the denominator, which is \( x \), and note that

\[
\frac{\sqrt{4x^2 + 1}}{x} = -\sqrt{\frac{4x^2 + 1}{x^2}} = -\sqrt{4 + \frac{1}{x^2}}.
\]

Since \( 1/x^2 \to 0 \) as \( x \to -\infty \), we conclude that

\[
\lim_{x \to -\infty} \left( \frac{x}{\sqrt{4x^2 + 1} - 3x} \right) \left( \frac{1/x}{1/x} \right) = \lim_{x \to -\infty} \frac{1}{-\sqrt{4 + \frac{1}{x^2}} - 3} = \frac{1}{-\sqrt{4} - 3} = -\frac{1}{5}.
\]
Long answer question — you must show your work

3. **4 marks** Compute the limit \( \lim_{x \to 1} \frac{1 - x}{\sqrt{4x + 5} - \sqrt{10} - x} \).

**Answer:** \(-\frac{6}{5}\)

**Solution:** If we try to do the limit naively we get \(0/0\). Hence we must simplify.

\[
\frac{1 - x}{\sqrt{4x + 5} - \sqrt{10} - x} = \frac{(1 - x)(\sqrt{4x + 5} + \sqrt{10} - x)}{(\sqrt{4x + 5} + \sqrt{10} - x)(\sqrt{4x + 5} - \sqrt{10} - x)}
\]

\[
= \frac{(1 - x)(\sqrt{4x + 5} + \sqrt{10} - x)}{(4x + 5) - (10 - x)}
\]

\[
= \frac{(1 - x)(\sqrt{4x + 5} + \sqrt{10} - x)}{5x - 5}
\]

\[
= \frac{(1 - x)(\sqrt{4x + 5} + \sqrt{10} - x)}{-5(1 - x)}
\]

\[
\rightarrow \frac{\sqrt{4x + 5} + \sqrt{10} - x}{-5}
\]

So the limit is

\[
\lim_{x \to 1} \frac{1 - x}{\sqrt{4x + 5} - \sqrt{10} - x} = \lim_{x \to 1} \frac{\sqrt{4x + 5} + \sqrt{10} - x}{-5} = \frac{\sqrt{4 + 5} + \sqrt{10 - 1}}{-5} = \frac{6}{-5} = -\frac{6}{5}
\]