The remainder of this page has been left blank for your workings.
Very short answer questions

1. 2 marks Each part is worth 1 marks. Please write your answers in the boxes.

(a) Compute \( \lim_{x \to -1} \sqrt{3x^3} + 7 \).

Answer: 2

Solution:

\[
\lim_{x \to -1} \sqrt{3x^3} + 7 = \sqrt{\lim_{x \to -1} (3x^3) + 7} \\
= \sqrt{3 \lim_{x \to -1} x^3 + 7} \\
= \sqrt{4} = 2.
\]

(b) Compute the limit \( \lim_{x \to 2} \frac{x^2 - 4}{x - 2} \).

Answer: 4

Solution: If try naively then we get \(0/0\), so we simplify first:

\[
\frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{(x - 2)} = x + 2
\]

Hence the limit is \( \lim_{x \to 2} (x + 2) = 4. \)
Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

(a) Find the left-hand and right-hand limits of \( \frac{|6 + 3x|}{x + 2} \) as \( x \to -2 \).

Answer: \( l = -3, r = +3 \)

Solution:

\[
\frac{|6 + 3x|}{x + 2} = 3 \cdot \frac{|x + 2|}{x + 2} = \begin{cases} 
3 \cdot \frac{-x - 2}{x + 2} = -3 & x < -2 \\
3 \cdot \frac{x + 2}{x + 2} = +3 & x > -2 
\end{cases}
\]

So left-hand limit is \(-3\) while right-hand limit is \(+3\).

(b) Evaluate \( \lim_{x \to -\infty} \frac{3x + 5}{x - \sqrt{x^2 - x + 5}} \)

Answer: \( \frac{3}{2} \)

Solution: We divide by the highest power of the denominator, which is \( x \) and note that

\[
\frac{\sqrt{x^2 - x + 5}}{x} = -\sqrt{\frac{x^2 - x + 5}{x^2}} = -\sqrt{\frac{1}{x} \cdot \frac{5}{x^2}}.
\]

Since \( 1/x \to 0 \) and also \( 1/x^2 \to 0 \) as \( x \to -\infty \), we conclude that

\[
\lim_{x \to -\infty} \frac{3x + 5}{x - \sqrt{x^2 - x + 5}} = \lim_{x \to -\infty} \frac{3 + \frac{5}{x}}{1 + \sqrt{1 - \frac{1}{x} + \frac{5}{x^2}}} = \frac{3}{1 + 1} = \frac{3}{2}.
\]
Long answer question — you must show your work

3. 4 marks Compute the limit \( \lim_{x \to 1} \frac{x - 1}{\sqrt{x} - \sqrt{2} - x} \).

Answer: 1

Solution: If we try to do the limit naively we get 0/0. Hence we must simplify.

\[
\frac{x - 1}{\sqrt{x} - \sqrt{2} - x} = \frac{x - 1}{\sqrt{x} - \sqrt{2} - x} \cdot \frac{\sqrt{x} + \sqrt{2} - x}{\sqrt{x} + \sqrt{2} - x}
\]

\[
= \frac{(x - 1)(\sqrt{x} + \sqrt{2} - x)}{x - (2 - x)}
\]

\[
= \frac{(x - 1)(\sqrt{x} + \sqrt{2} - x)}{2x - 2}
\]

\[
= \frac{\sqrt{x} + \sqrt{2} - x}{2}
\]

So the limit is

\[
\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - \sqrt{2} - x} = \lim_{x \to 1} \frac{\sqrt{x} + \sqrt{2} - x}{2}
\]

\[
= \frac{1 + 1}{2}
\]

\[
= 1.
\]
The remainder of this page has been left blank for your workings.
Very short answer questions

1. [2 marks] Each part is worth 1 marks. Please write your answers in the boxes.

(a) Compute $\lim_{x \to 2} \frac{1}{\sqrt{1 + 4x}}$.

Answer: $\frac{1}{3}$

Solution:

$$\lim_{x \to 2} \frac{1}{\sqrt{1 + 4x}} = \frac{1}{\sqrt{\lim_{x \to 2} (1 + 4x)}}$$

$$= \frac{1}{\sqrt{1 + 4 \lim_{x \to 2} x}}$$

$$= \frac{1}{\sqrt{9}} = \frac{1}{3}$$

(b) Compute the limit $\lim_{x \to 4} \frac{x - 4}{16 - x^2}$.

Answer: $-\frac{1}{8}$

Solution: If try naively then we get $0/0$, so we simplify first:

$$\frac{x - 4}{16 - x^2} = \frac{x - 4}{(4 - x)(4 + x)} = -\frac{1}{x + 4}$$

Hence the limit is $\lim_{x \to 4} \left(-\frac{1}{x + 4}\right) = -\frac{1}{8}$. 
Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.

(a) Compute both one-sided limits \( \lim_{x \to -3^+} \frac{|x + 3|}{2x + 6} \) and \( \lim_{x \to -3^-} \frac{|x + 3|}{2x + 6} \).

Answer: \( r = +1/2, l = -1/2 \)

Solution:

\[
\frac{|x + 3|}{2x + 6} = \frac{|x + 3|}{2(x + 3)} = \begin{cases} 
\frac{-x - 3}{2(x + 3)} = -\frac{1}{2} & x < -3 \\
\frac{x + 3}{2(x + 3)} = \frac{1}{2} & x > -3
\end{cases}
\]

So left-hand limit is \(-1/2\) while right-hand limit is \(+1/2\).

(b) Evaluate \( \lim_{x \to -\infty} \frac{\sqrt{x^2 + 3} - x}{2x + 1} \)

Answer: \(-1\)

Solution: We divide by the highest power of the denominator, which is \( x \) and note that when \( x < 0 \)

\[
\frac{\sqrt{x^2 + 3}}{x} = -\sqrt{\frac{x^2 + 3}{x^2}} = -\sqrt{1 + \frac{3}{x^2}}.
\]

Since \( 1/x \to 0 \) and also \( 1/x^2 \to 0 \) as \( x \to -\infty \), we conclude that

\[
\lim_{x \to -\infty} \frac{\sqrt{x^2 + 3} - x}{2x + 1} = \lim_{x \to -\infty} \frac{-\sqrt{1 + \frac{3}{x^2}} - 1}{2 + 1/x} = \frac{-1 - 1}{2} = -1.
\]
Long answer question — you must show your work

3. **4 marks** Compute the limit \( \lim_{x \to 1} \frac{x - 1}{\sqrt{2x - 1} - \sqrt{x}} \).

\[ \text{Answer: 2} \]

**Solution:** If we try to do the limit naively we get \(0/0\). Hence we must simplify.

\[
\frac{x - 1}{\sqrt{2x - 1} - \sqrt{x}} = \frac{x - 1}{\sqrt{2x - 1} - \sqrt{x}} \cdot \frac{\sqrt{2x - 1} + \sqrt{x}}{\sqrt{2x - 1} + \sqrt{x}}
\]

\[
= \frac{(x - 1)(\sqrt{2x - 1} + \sqrt{x})}{(2x - 1) - x}
\]

\[
= \frac{(x - 1)(\sqrt{2x - 1} + \sqrt{x})}{x - 1}
\]

\[
= \sqrt{2x - 1} + \sqrt{x}
\]

So the limit is

\[
\lim_{x \to 1} \frac{x - 1}{\sqrt{2x - 1} - \sqrt{x}} = \lim_{x \to 1} \left( \sqrt{2x - 1} + \sqrt{x} \right)
\]

\[= 2. \]
First Name: ____________________________ Last Name: ____________________________

Student-No: ____________________________ Section: ____________________________

Grade: ____________________________

The remainder of this page has been left blank for your workings.
Very short answer questions

1. [2 marks] Each part is worth 1 marks. Please write your answers in the boxes.

(a) Compute \( \lim_{x \to 2} \sqrt{x^2 + 6x} \).

Answer: 4

**Solution:**

\[
\lim_{x \to 2} \sqrt{x^2 + 6x} = \sqrt{\lim_{x \to 2}(x^2 + 6x)} = \sqrt{(\lim_{x \to 2} x)^2 + 6 \lim_{x \to 2} x} = \sqrt{4 + 12} = \sqrt{16} = 4.
\]

(b) Compute the limit \( \lim_{x \to -1} \frac{x^2 - 2x - 3}{x + 1} \).

Answer: -4

**Solution:** If try naively then we get 0/0, so we simplify first:

\[
\frac{x^2 - 2x - 3}{x + 1} = \frac{(x + 1)(x - 3)}{(x + 1)} = x - 3 \quad \text{for } x \neq -1
\]

Hence the limit is \( \lim_{x \to -1} (x - 3) = -4 \).
Name: ___________________________ Student-No: ___________________________

**Short answer questions — you must show your work**

2. **4 marks** Each part is worth 2 marks.

   (a) Find the left-hand and right-hand limits of \( \frac{\sqrt{(x-1)^2}}{x-1} \) as \( x \to 1 \).

   **Answer:** \( l = -1, r = +1 \)

   **Solution:**

   \[
   \frac{\sqrt{x^2 - 2x + 1}}{x - 1} = \frac{\sqrt{(x-1)^2}}{x-1} = \begin{cases} 
   \frac{1-x}{x-1} = -1 & x < 1 \\
   \frac{x-1}{x-1} = +1 & x > 1
   \end{cases}
   \]

   So left-hand limit is \(-1\) while right-hand limit is \(+1\).

   (b) Evaluate \( \lim_{x \to -\infty} \frac{x - \sqrt{x^2 + 4}}{2x + 5} \)

   **Answer:** \( 1 \)

   **Solution:** We divide by the highest power of the denominator, which is \( x \) and note that

   \[
   \frac{\sqrt{x^2 + 4}}{x} = -\sqrt{\frac{x^2 + 4}{x^2}} = -\sqrt{1 + \frac{4}{x^2}} \quad \text{for} \quad x < 0
   \]

   Since \( 1/x \to 0 \) and also \( 1/x^2 \to 0 \) as \( x \to -\infty \), we conclude that

   \[
   \lim_{x \to -\infty} \frac{x - \sqrt{x^2 + 4}}{2x + 5} = \lim_{x \to -\infty} \frac{1 - \left( -\sqrt{1 + \frac{4}{x^2}} \right)}{2 + \frac{5}{x}} = \frac{1 + 1}{2} = \frac{2}{2}.
   \]
Long answer question — you must show your work

3. 4 marks Compute the limit $\lim_{x \to 3} \frac{x - 3}{\sqrt{x + 1} - \sqrt{7} - x}$. 

Answer: 2

Solution: If we try to do the limit naively we get $0/0$. Hence we must simplify.

$$\frac{x - 3}{\sqrt{x + 1} - \sqrt{7} - x} = \frac{x - 3}{\sqrt{x + 1} - \sqrt{7} - x} \cdot \frac{\sqrt{x + 1} + \sqrt{7} - x}{\sqrt{x + 1} + \sqrt{7} - x} = \frac{(x - 3)(\sqrt{x + 1} + \sqrt{7} - x)}{(x + 1) - (7 - x)} = \frac{(x - 3)(\sqrt{x + 1} + \sqrt{7} - x)}{2x - 6} = \frac{\sqrt{x + 1} + \sqrt{7} - x}{2} \quad \text{for } x \neq 3$$

So the limit is

$$\lim_{x \to 3} \frac{x - 3}{\sqrt{x + 1} - \sqrt{7} - x} = \lim_{x \to 3} \frac{\sqrt{x + 1} + \sqrt{7} - x}{2} = \frac{2 + 2}{2} = 2.$$
The remainder of this page has been left blank for your workings.
Very short answer questions

1. [2 marks] Each part is worth 1 marks. Please write your answers in the boxes.

(a) Compute \( \lim_{x \to 1} \sqrt{x} + 4x^2 \).

\[ \text{Solution:} \]
\[
\lim_{t \to 1} \sqrt{x} + 4x^2 = \sqrt{\lim_{t \to 1} (x + 4x^2)} \\
= \sqrt{\lim_{t \to 1} x + 4 \lim_{t \to 1} x^2} \\
= \sqrt{5}.
\]

(b) Compute the limit \( \lim_{x \to -2} \frac{x^2 - 4}{x + 2} \).

\[ \text{Solution:} \] If try naively then we get 0/0, so we simplify first:
\[
\frac{x^2 - 4}{x + 2} = \frac{(x - 2)(x + 2)}{(x + 2)} = x - 2
\]

Hence the limit is \( \lim_{x \to -2} (x - 2) = -4 \).
Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.

(a) Find the left-hand and right-hand limits of \( \frac{6-2x}{|x-3|} \) as \( x \to 3 \).

Answer: \( l = 2, r = -2 \)

Solution:

\[
\frac{6 - 2x}{|x - 3|} = 2 \cdot \frac{3 - x}{|x - 3|} = \begin{cases} 
2 \cdot \frac{3 - x}{3 - x} & x < 3 \\
2 \cdot \frac{3 - x}{x - 3} & x > 3
\end{cases}
\]

So left-hand limit is 2 while right-hand limit is -2.

(b) Evaluate \( \lim_{x \to -\infty} \frac{\sqrt{x^2 + 5} - x}{x + 1} \)

Answer: -2

Solution: We consider only \( x < 0 \), since \( x \to \infty \). Note that

\[
\sqrt{x^2 + 5} = \sqrt{x^2 \sqrt{1 + 5/x^2}} = |x| \sqrt{1 + 5/x^2} = -x \sqrt{1 + 5/x^2}
\]

since \( |x| = -x \) for \( x < 0 \). Since \( 1/x \to 0 \) and also \( 1/x^2 \to 0 \) as \( x \to -\infty \), we conclude that

\[
\lim_{x \to -\infty} \frac{\sqrt{x^2 + 5} - x}{x + 1} = \lim_{x \to -\infty} \frac{-x \sqrt{1 + 5/x^2} - x}{x + 1} = \lim_{x \to -\infty} \frac{-\sqrt{1 + 5/x^2} - 1}{1 + 1/x} = -2.
\]
Long answer question — you must show your work

3. 4 marks Compute \( \lim_{x \to 2} \frac{x - 2}{\sqrt{x - 1} - \sqrt{3 - x}} \).

Answer: 1

**Solution:** If we try to do the limit naively we get \(0/0\). Hence we must simplify.

\[
\frac{x - 2}{\sqrt{x - 1} - \sqrt{3 - x}} = \frac{x - 2}{\sqrt{x - 1} - \sqrt{3 - x}} \cdot \frac{\sqrt{x - 1} + \sqrt{3 - x}}{\sqrt{x - 1} + \sqrt{3 - x}}
\]

\[
= \frac{(x - 2)(\sqrt{x - 1} + \sqrt{3 - x})}{(x - 1) - (3 - x)}
\]

\[
= \frac{(x - 2)(\sqrt{x - 1} + \sqrt{3 - x})}{2x - 4}
\]

\[
= \frac{\sqrt{x - 1} + \sqrt{3 - x}}{2}
\]

So the limit is

\[
\lim_{x \to 2} \frac{x - 2}{\sqrt{x - 1} - \sqrt{3 - x}} = \lim_{x \to 2} \frac{\sqrt{x - 1} + \sqrt{3 - x}}{2} = \frac{1 + 1}{2} = 1
\]