1. \( F(\theta) = \frac{\mu W}{\mu \sin \theta + \cos \theta} \), \( \mu, W > 0 \), \( \theta \) between \( 0 \) and \( \frac{\pi}{2} \).

\[
F'(\theta) = \mu W (-1) \left( \mu \sin \theta + \cos \theta \right)^{-2} \left( \mu \cos \theta - \sin \theta \right)
\]

\[
= \frac{-\mu^2 W \cos \theta + \mu W \sin \theta}{(\mu \sin \theta + \cos \theta)^2}
\]

- So \( F'(\theta) = 0 \) when

\[
-\mu^2 W \cos \theta + \mu W \sin \theta = 0.
\]

\[
\sin \theta = \mu \cos \theta,
\]

\[
\tan \theta = \mu,
\]

\( \theta = \arctan \mu \).

- Note that \( F(\theta) = \frac{\mu W}{\mu \cos \theta + \cos \theta} = \mu W \).

- Note that when \( \mu \sin \theta + \cos \theta = 0 \),

i.e. \( \tan \theta = -\frac{1}{\mu} \), \( \theta = \arctan(-\frac{1}{\mu}) + \pi \) (if \( \theta < \frac{\pi}{2} \)).
is undefined. In fact, \( F(\Theta) \to +\infty \) as

\[ \Theta \to \arctan \left( \frac{-1}{\mu} \right) + \pi. \]

- \( \Theta \) shouldn’t be bigger than \( \pi/2 \), anyway.

\[ -F''(\Theta) = \text{Thus, if } F(\arctan \mu) \text{ is smaller} \]

\[ \text{than } \mu W = F(0), \text{ } \Theta = \arctan \mu \text{ will be the global minimum:} \]

\[ F(\arctan \mu) = \frac{\mu W}{\mu \sin(\arctan \mu) + \cos(\arctan \mu)} \]

\[ \sqrt{\mu^2 + 1} \]

\[ \mu \text{ and } \tan \Theta = \frac{\mu}{1} \]

\[ \sin \Theta = \frac{\mu}{\sqrt{\mu^2 + 1}} \text{ and } \cos \Theta = \frac{1}{\sqrt{\mu^2 + 1}}. \]
Thus

$$F(\arctan \mu) = \frac{\mu W}{\frac{\mu^2}{\sqrt{\mu^2 + 1}} + \frac{1}{\sqrt{\mu^2 + 1}}}$$

$$= \frac{\mu W \sqrt{\mu^2 + 1}}{\mu^2 + 1}$$

$$= \frac{\mu W}{\sqrt{\mu^2 + 1}} < \mu W = F(0),$$

since $\sqrt{\mu^2 + 1} > 1$.

2. (a). If $H=0$, the graph of $\frac{dP}{dt}$ as a function of $P$ is a parabola with intercepts at $P=0$ and $P=K$:

![Graph of dp/dt versus P](image)
(b) If \( 0 < H < \frac{rk^2}{4} \), the graph above is shifted down by \( H \), but not so far as to not have roots:

\[
\frac{dp}{dt} = p
\]

The roots satisfy

\[
0 = rP(K - p) - H = -rP^2 + rkP - H
\]

which is quadratic in \( P \). The roots are

\[
P = K \pm \sqrt{K^2 - 4H} \quad \frac{r}{2}
\]

(c). \( P_1 = K - \sqrt{\frac{K^2 - 4H}{r}} \) is unstable

and \( P_2 = K + \sqrt{\frac{K^2 - 4H}{r}} \) is stable.
Both populations are said to be at equilibrium since $\frac{dp}{dt} = 0$ when the population is at one of these roots ($\text{no change in } P \text{ over time}$).

$P_1$ is unstable because if $P$ starts close to $P_1$, $P$ grows or shrinks. $P_2$ is stable because $P$ tends toward $P_2$ over time.

\[
\begin{align*}
\text{Start here } \cdots \\
\frac{dp}{dt} < 0 \Rightarrow P \text{ shrinks } \Rightarrow P \text{ moves away from } P_1, \\
\text{start here, } \frac{dp}{dt} > 0, P \text{ tends to } P_2.
\end{align*}
\]

(d) If $K > \frac{rk^2}{4}$, the entire parabola is shifted below the axis, so $\frac{dp}{dt} < 0$ for all $P$, $\Rightarrow$ extinction!