There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the correctness and coherence of your solutions, as well as on their elegance. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed at the front of the lecture hall before the start of class on Tuesday, Nov 21. The online assignment will close at 8:00 on Tuesday, Nov 21.

1. An object of weight $W$ is dragged along a horizontal plane by means of force $F$ whose line of action makes an angle of $\theta$ with the plane. The magnitude of the force is given by

\[ F = \frac{\mu W}{\mu \sin \theta + \cos \theta} \]

where $\mu$ is the coefficient of friction. For what value of $\theta$ is it easiest to pull the object?

2. The logistic differential equation describes the rate of change of a population $P$ over time:

\[ \frac{dP}{dt} = rP(K - P), \]

where $r$ and $K$ are positive constants describing the natural growth rate and carrying capacity of the population, respectively. If the population is harvested at a constant rate $H$, the differential equation can be adjusted to describe this:

\[ \frac{dP}{dt} = rP(K - P) - H. \]

(a) Sketch a graph of $\frac{dP}{dt}$ as a function of $P$, assuming that $H = 0$.

(b) Sketch a graph of $\frac{dP}{dt}$ as a function of $P$, assuming that $0 < H < \frac{rK^2}{4}$.

(c) Assuming that $0 < H < \frac{rK^2}{4}$, there is one starting population that is said to be at stable equilibrium, and one starting population that is consider to be at unstable equilibrium. Give expressions for these populations in terms of $r$, $K$, and $H$, and describe in one or two short paragraphs why one is said to be stable and the other is said to be unstable.

(d) What happens to the population over time if $H > \frac{rK^2}{4}$?