Given the testimony of the witness,

In the accident was involved a blue car.

What is the color of the car involved?

An eye witness identifies the car as blue.

The colors: blue or green.

A car was involved in a hit-and-run accident at night. Cases have one of the above.

None were exercised.

Bayes' formula.

Use of data, please.
We're looking for: \( P(E|L) \).

\[
P(E|L) = 95\% \\
P(F|L) = 9\%
\]

\[
P(E) = \frac{2 + 18}{2} = 10
\]

Problem data:

\[ P = \frac{1}{4} \text{ witness testified to seeing a blue car} \]

\[ E = \frac{1}{6} \text{ the car was involved in the accident} \]

\[ F = \frac{3}{4} \text{ the car was not involved in the accident} \]

Solution:

Define events:

- \( E \): Witness reliability: 95% accurate.
- \( F \): Witness reliability: 9% accurate.
\[ P(E|L) = \frac{18}{66} = 27 \% \]

\[ P(E|\overline{L}) = \frac{5}{9} = 55.5 \% \]

\[ \text{Correctly identify blue cases in green cases.} \]

\[ 97 \% \text{ accurate} \]

\[ P(L|E) = 1 - P(\overline{L}|E) \]

\[ \frac{50}{50} = 100 \% \]

\[ \text{Law of Total Probability:} \]

\[ P(E) = P(E|L) \cdot P(L) + P(E|\overline{L}) \cdot P(\overline{L}) \]

\[ P(E) = P(E|L) \cdot P(L) \]

\[ P(E|L) \cdot P(L) = P(E|\overline{L}) \cdot P(\overline{L}) \]

\[ \text{Bayes' Formula:} \]
Exercise: (Gambler's ruin)

Consider the following game:

- an initial pool of money: \( N = $100 \)
- initially you have \( n = $10 \) of that.

The following is repeated until one of the parties has all the money.

Flip a fair coin

- If Heads: you gain $1
- If Tails: bank loses $1

Q: What is the prob. of winning?
Denote \( a_n = P(E_n) \).

\( E_n = \{ n \in \mathbb{N} \mid n = 0 \} \)

\( F = \{ \text{the first flip is Heads} \} \)

Now we use the law of large prob.

Game over.

Stage 2: Remaining Flips until

Stage 1: First Flip

We view the game as a two-step experiment:

\( \text{Sol} \)
Then
\[ P(\text{En}) = \frac{P(\text{En} \mid F) \cdot P(F)}{\frac{P(\text{En} \mid F^c) \cdot P(F^c)}{\frac{P(\text{En+1})}{2}} + \frac{P(\text{En-1})}{2}} \]

\[ P(\text{En} \mid F) = P(\text{win, starting from } \#n \mid \text{first flip is H}) \]
\[ = P(\text{win, starting from } \#(n+1)) \]
\[ = P(\text{En+1}) \]

\[ a_n = \frac{1}{2} a_{n+1} + \frac{1}{2} a_{n-1} \]
\[
\frac{N}{n} = a_n \\
\text{Thus, } a_n = a_{n-1} - q_{n-2} = \ldots = a_3 - q_2 = a_2 - q_1 = a_1 - q_0 = a_0 = A_0 = N \frac{0}{1} = \frac{N}{1} = 1
\]