1. A die is rolled until the first time the result is 6. Let $X$ be the number of rolls. Next, we choose a sample, with replacement, of size $X$ from an urn with 9 red and 3 green balls. Let $Y$ be the number of green balls in the sample.
   (a) Compute the conditional p.m.f. $P_{Y|X}(y|x)$.
   (b) Compute $E[Y|X]$.
   (c) Use your answer from part (b) to compute $E(Y)$.

2. Suppose that the random variables $X, Y$ have joint density function
   \[ f(x, y) = \begin{cases} \frac{x+4y}{2} & \text{if } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq \frac{1}{2}, \\ 0 & \text{otherwise}. \end{cases} \]
   (a) What is the conditional p.d.f. of $Y$ given that $X = 1$?
   (b) Find the conditional expectation $E(Y \mid X = 1)$.

3. Let $X$ and $Y$ be two independent uniform random variables on $(0, 1)$.
   (a) Using the convolution formula, find the p.d.f. of the random variable $Z = X + Y$, and graph it.
   (b) What is the moment generating function of $Z$?

4. Suppose that $X$ has moment generating function
   \[ M_X(t) = \frac{1}{3} + \frac{1}{2} e^{-t} + \frac{1}{6} e^{2t}. \]
   (a) Find the mean and variance of $X$ by differentiating the m.g.f. above.
   (b) Find the p.m.f. of $X$. Use your expression for the p.m.f. to check your answers from part (a).

5. Let $X_n \sim \text{Bin}(n, \frac{\lambda}{n})$ for some $\lambda > 0$. Show that the moment generating function $M_{X_n}(t)$ converges as $n \to \infty$, and show that the limit is the m.g.f. of the Poisson($\lambda$) random variable.