1. We toss two dice. Consider the events

\[ E = \text{“The sum of the outcomes is odd”} \]
\[ F = \text{“At least one outcome is 4”} \]

Calculate the conditional probabilities \( P(E \mid F) \) and \( P(F \mid E) \).

2. Let \( X \) be a geometric random variable with a given parameter \( 0 < p < 1 \). Show that for all integers \( m, n \geq 1 \) we have

\[ P(X = n + m \mid X > n) = P(X = m) \]

and

\[ P(X > n + m \mid X > n) = P(X > m). \]

(For this reason, we say that a geometric random variable has no memory.)

3. A fair die is rolled repeatedly.
   (a) Give an expression for the probability that the first five rolls give a three at most two times.
   (b) Calculate the probability that the first three does not appear before the fifth roll.
   (c) Calculate the probability that the first three appears before the twentieth roll, but not before the fifth roll.

4. Let \( m \) be an integer chosen uniformly from \( \{1, \ldots, 100\} \). Decide whether the following events are independent:
   (a) \( E = \{m \text{ is odd}\} \) and \( F = \{m \text{ is divisible by 7}\} \)
   (b) \( E = \{m \text{ has two digits}\} \) and \( F = \{m \text{ is divisible by 3}\} \)
   (c) \( E = \{m \text{ is prime}\} \) and \( F = \{\text{one of the digits of } m \text{ is a 4}\} \)

5. Draw 5 cards at random from a standard deck of 52 cards. Find
   (a) \( P(\text{full house} \mid \text{full-house or three-of-a-kind}) \).
   (b) \( P(\text{full house} \mid \text{at least two aces}) \).
   (c) \( P(\text{at least two aces} \mid \text{full-house}) \).

6. Three buses are stuck at a traffic jam. On first bus there are 24 men and 12 women, on the second 20 men and 15 women, and on the third 14 men and 21 women. A uniformly chosen bus is picked, and a uniformly random passenger from that bus is selected. What is the probability that the selected passenger is a man?

**Extra practice problems** (do not hand in):
Chapter 1: 16, 18, 19
Chapter 2: 5, 8, 13, 18