1. Let $X$ be a geometric random variable with parameter $\frac{1}{5}$.
   (a) Use Chebyshev's inequality to find an upper bound for $P(X \geq 15)$.
   (b) Compute $P(X \geq 15)$ exactly and compare with the result in (a).

2. In a lottery, you can win a nice smartphone ($1200). Each ticket costs $20, and 4\% of tickets are a win. You decide to buy tickets until you win the phone.
   (a) Give a lower bound on the probability that this is better than just buying the phone in store.
   (b) Give a price that your costs can exceed with probability at most 25\%.

3. Copying 1 billion ($10^9$) bits from a USB drive to your hard disk takes about 0.2 seconds under ideal conditions. However, every bit has a probability of roughly $p = 10^{-8}$ to be copied incorrectly. Using error correcting codes, your PC is able to recognize and correct flawed bits during the copying process. Assume that it takes 0.02 seconds to correct a flawed bit. We are interested in the probability that copying a movie of 2.5 gigabytes (a byte is 8 bits, and giga is a billion) takes less than 9 seconds.
   (a) Write down a precise expression for this probability.
   (b) Give a lower bound on this probability.
   For the next two questions, use the Poisson approximation for the number of flawed bits.
   (c) Give a lower bound on this probability using Chebyshev's inequality.
   (d) Compute this probability.

4. Let $X$ be a Poisson random variable with parameter $\lambda$
   a) Which $n = n(\lambda) \geq 0$ is the most likely value of $X$, i.e. maximizes $P(X = n)$?
   b) Suppose the experiment described by $X$ has returned the value $n \geq 0$. Which parameter $\lambda = \lambda(n)$ maximizes $P(X = n)$?