1. Suppose that the continuous RV $X$ has c.d.f. given by

$$F(x) = \begin{cases} 
0 & \text{if } x < \frac{1}{\sqrt{2}} \\
5 - 12\sqrt{2}x + 18x^2 - 4\sqrt{2}x^3 & \text{if } \frac{1}{\sqrt{2}} \leq x < \sqrt{2} \\
1 & \text{if } \sqrt{2} \leq x 
\end{cases}$$

(a) Find the smallest interval $[a, b]$ such that $P(a \leq X \leq b) = 1$.
(b) Find $P(0 < X < \frac{1}{2})$.
(c) Find $P(X = 1)$.
(d) Find $P(1 \leq X \leq \frac{3}{2})$.
(e) Find the p.d.f. of $X$.

2. Define the function

$$f(x) = \begin{cases} 
9x^2 - 4x^3 + b & x \in [0, 1] \\
0 & \text{otherwise}
\end{cases}$$

Show that there is no value of $b$ for which this is the p.d.f. of some continuous RV.

3. Suppose a continuous RV $X$ has the p.d.f.

$$f(x) = \begin{cases} 
c & x > 0 \\
\frac{c}{1+x^2} & x \leq 0
\end{cases}$$

(a) Find the c.d.f. of $X$.
(b) What must be the value of $c$?
(c) Find $\mathbb{E}X$.
(d) Compute $\mathbb{E}\left(\frac{1}{\sqrt{1+X}}\right)$.

4. Let $X$ be a random variable with p.d.f.

$$f(x) = \begin{cases} 
2x^{-2} & x > 2 \\
0 & \text{otherwise}
\end{cases}$$

(a) Compute the c.d.f. of $X$.
(b) Find $P(X > 3)$.
(c) Find $P(X > 3 | X < 5)$.
(d) Calculate $\mathbb{E}\sqrt{X}$

6. Let $c > 0$ and $X \sim \text{Unif}[0, c]$. Show that the RV $Y = c - X$ has the same c.d.f. and therefore also the same p.d.f. as $X$.

7. A stick of length $\ell$ is broken into two pieces at a position $X \sim \text{Unif}[0, \ell]$. Let $Y$ denote the length of the smaller piece.
(a) Calculate the c.d.f. of $Y$, that is, calculate $P(Y \leq b)$.
(b) Calculate the p.d.f. of $Y$. Can you identify what kind of random variable $Y$ is?

8. Let $X$ be an Exp(4) random variable. Find a number $a$ such that $\{X \in [0, 1]\}$ is independent of $\{X \in [a, 2]\}$

9. (a) Suppose that the duration $T$ (in hours) of your morning routine (breakfast, shower, etc.) is modeled by an exponential RV with parameter $\lambda$. You set your alarm 1 hour before your bus leaves for UBC. For which value of $\lambda$ do you have a 50% chance of catching the bus?
(b)* Calculate the $n$th moment of $T$. 
