1. Let $X$ be a r.v. taking values in $\{1, \ldots, 6\}$ with p.m.f. of the form
   \[ P(X = k) = ck. \]
   (a) Find $c$.
   (b) Find the probability that $X$ is odd.

2. Let $X$ be a r.v. taking values in $\{1, 2, \ldots\}$ with p.m.f. of the form
   \[ P(X = k) = \frac{c}{k(k + 1)}. \]
   (a) Show that $P(X \geq k) = c/k$ for any $k \geq 1$.
   (b) Find $c$.
   (c)* Find the probability that $X$ is odd.

3. In a card game, 13 cards are given to you out of a deck of 52. This game is being played 1000 times. Identify (with names and parameters) the following random variables:
   (a) The number of games in which all cards you receive have the same suit.
   (b) The first time where the number of aces you receive is at least 1.
   (c) The number of games in which you receive exactly three aces.
   (d) The third time in which you received no aces.

4. Let $X$ be a geometric random variable with a given parameter $0 < p < 1$. Show that for all integers $m, n \geq 1$ we have
   \[ P(X = n + m \mid X > n) = P(X = m) \]
   and
   \[ P(X > n + m \mid X > n) = P(X > m). \]
   (For this reason, we say that a geometric random variable has no memory.)

5. Let $X$ be a discrete random variable which has no memory, i.e., it satisfies
   \[ P(X = n + m \mid X > n) = P(X = m) \]
   for all $m, n \geq 1$. Show that $X$ has to be a geometric random variable with parameter $p = P(X = 1)$. That is, show that the p.m.f. of $X$ is $P(X = k) = p(1 - p)^{k-1}$, with $p$ as just stated. \textit{Hint:} Use $P(X = k) = P(X = k + 1 \mid X > 1)$ repeatedly.

6. An oil company conducts a geological study that indicates that an exploratory oil well should have a 20% chance of striking oil.
   (a) What is the probability that the first strike comes on the third well drilled?
   (b) What is the probability that the third strike comes on the seventh well drilled?