Math 302, Assignment 1  

Due Sep. 21

1. Let $S = \{1, \emptyset, c\}$ be a sample space. List all possible events.

2. Your baking cupboard contains 1 cup of whole wheat flour, 1 cup of white flour, 1 cup of brown sugar, 1 cup of white sugar, and 2 eggs. Consider the following random baking experiment: You thoroughly mix three randomly chosen ingredients in a bowl and throw it into the oven.
   (a) Write down the sample space of this experiment.
   (b) What is the probability that you will actually bake a cake? (A super basic sponge cake, anyway).

3. Let $S$ be a sample space and $\mathbb{P}$ be a probability. Prove that there can’t exist events $E, F$ that satisfy
   \[ \mathbb{P}(E \setminus F) = \frac{1}{3}, \; \mathbb{P}(E \cap F) = \frac{1}{4}, \text{ and } \mathbb{P}(E^c \cap F^c) = \frac{1}{2}. \]

4. Let $A, B, C$ be events in a sample space $S$. Prove that
   (a) $\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B),$
   (b) $\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(A \cap B) - \mathbb{P}(A \cap C) - \mathbb{P}(B \cap C) + \mathbb{P}(A \cap B \cap C).$

5. Assuming a fair poker deal, what is the probability of a
   (a) royal flush
   (b) straight flush
   (c) flush
   (d) straight
   (e) two pair
   See https://en.wikipedia.org/wiki/List_of_poker_hands for the definition of these poker hands.

6. How many different anagrams (rearrangements of letters) can be formed from the letters of COMBINATORICS?

7. We toss a fair die four times. What is the probability that all tosses produce different outcomes?

8*. Prove that the number of unordered sequences of length $k$ with elements from a set $X$ of size $n$ is \( \binom{n+k-1}{k} \).
   Hint: For illustration, first consider the example $n = 4, k = 6$. Let the 4 elements of the set $X$ be denoted $a, b, c, d$. Argue that any unordered sequence of size 6 consisting of elements $a, b, c, d$ can be represented uniquely by a symbol similar to “$\cdot | \cdot | \cdot | \cdot$”, corresponding to the sequence $aabced$. Now count the number of choices for the vertical bars.