Fall 2009. Math 606:102. Topics in Differential Equations — Optimal transportation

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In this course, we will go over basics of optimal transportation theory, one of the most active research areas in analysis and partial differential equations. This theory is based on the following variational problem, first initiated by Monge in 19's century: What is the most efficient way of moving a mass distribution to another, while one needs to pay cost for the transportation? (For example, consider the problem of matching water resources to residential areas.) This simple looking problem is a starting point of a lot of mathematical developments, from nonlinear partial differential equations to economics. In this course, a focus will be on its connections to the theory of partial differential equations and geometric/functional inequalities. We will emphasize understanding ideas rather than rigorous treatments. If time permits, we will cover a few recent progress in the field, where the choice of topics will be discussed as the course goes along.

Topics will include:

- Basic existence/uniqueness and characterization of optimal transportation: Kantorovich duality. Cyclical monotonicity. Geometry of optimal transportation
- Brenier's polar factorization theorem
- A PDE aspect of the theory: Monge-Ampère equations
- Geometry on the space of probability measures: displacement convexity
- Geometric/functional inequalities
- Evolution equations as gradient flows

List of further topics (tentative):

- Regularity of optimal transportation.
- Applications to meteorology: semi-geostropic equations
- Applications to economics

Prerequisite:

Working knowledge of first year graduate level real analysis. Some knowledge of partial differential equations will be helpful.

Main references:

- Two books by Cedric Villani
  - Topics in Optimal Transportation, AMS: this is a highly recommended textbook, but not required.
  - Optimal Transport: old and new. Springer. Available online at `http://www.umpa.ens-lyon.fr/~cvillani/Cedrif/B07C.StFlour.pdf`
- Additional reference.
  Ambrosio, Gigli and Savare: Gradient Flows (in Metric Spaces and in the Space of Probability Measures). Birkhauser.