Math 316 Midterm Exam II
March 18, 2009
Duration: 50 minutes

Last Name: ____________ First Name: ____________ Student Number: ________

Do not open this test until instructed to do so! This exam should have 7 pages, including this cover sheet. It is a closed book exam; no textbooks, calculators, laptops, formula sheets or other aids are allowed. Turn off any cell phones, pagers, etc. that could make noise during the exam. You must remain in this room until you have finished the exam. **Circle your solutions! Reduce your answer as much as possible. Explain your work. Relax.** Use the extra pages if necessary.

Read these UBC rules governing examinations:

(i) Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.

(ii) Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

(iii) No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

(iv) Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

- Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
- Speaking or communicating with other candidates.
- Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

(v) Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Problem	**Out of**	**Score**
1	20	
2	20	
Total	40	

Problem 1 (20 points)

Consider the heat conduction problem:

$$\begin{aligned} &u_t = u_{xx} - 4u, \quad 0 < x < 1,\ t > 0, \\ &u(0,t) = \alpha, \quad u(1,t) = \beta, \\ &u(x,0) = f(x) = \frac{1}{2}x. \end{aligned} \tag{1}$$

a) **[5]** Find the Fourier sine series of $f(x)$.

b) **[5]** For boundary data $\alpha = 0$ and $\beta = 1$, find the steady-state solution $v(x)$ of (1).

c) **[10]** For boundary data $\alpha = 0$ and $\beta = 0$, find the solution $u(x,t)$ of (1).

Extra page (Problem 1)

Extra page (Problem 1)

Problem 2 (20 points)

Consider the heat conduction problem:

$$\begin{aligned} &u_t = u_{xx}, \quad 0 < x < \pi,\ t > 0, \\ &u_x(0,t) = \alpha, \quad u_x(\pi,t) = \beta, \\ &u(x,0) = \cos(2x). \end{aligned} \tag{2}$$

a) [**10**] For boundary data $\alpha = 1$ and $\beta = 0$, find a particular solution $v(x,t)$ of (2) that satisfies the differential equation and the boundary conditions, but not necessarily the initial condition.

b) [**10**] For boundary data $\alpha = 0$ and $\beta = 0$, find the solution $u(x,t)$ of (2).

Extra page (Problem 2)

Extra page (Problem 2)