

Math 257/316, Midterm 1, Section 104

17 October 2007

**Instructions.** The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.

Maximum score 100.

1. Let  $f(x) = x$  on the interval  $0 < x < \pi$ .

(a) Sketch the even periodic extension of  $f$  with period  $2\pi$ .

[5 marks]

(b) Expand  $f(x)$  in a Fourier cosine series.

[15 marks]

**Hint:** It may be useful to know that

$$\int_0^{\pi} x \cos(nx) dx = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{-2}{n^2} & \text{if } n \text{ is odd} \end{cases}$$

2. Apply the method of separation of variables to determine the solution to the one dimensional heat equation with the following mixed boundary conditions:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$

$$\text{BC} : \frac{\partial u(0, t)}{\partial x} = 0 = u(\pi, t)$$

$$\text{IC} : u(x, 0) = 3 \cos\left(\frac{5}{2}x\right)$$

[40 marks]

3. Consider the second order differential equation:

$$Ly = 4x^2 y'' - (x^2 + x)y' + y = 0 \tag{1}$$

(a) Classify the points  $x \geq 0$  (do not include the point at infinity) as ordinary points, regular singular points, or irregular singular points.

[5 marks]

(b) Use the appropriate series expansion about the point  $x = 0$  to determine two independent solutions to (1). You only need to determine the first three non-zero terms in each case.

[35 marks]