Math 257/316, Midterm 1, Section 103
11 October 2006

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.

Maximum score 100.

1. Apply the method of separation of variables to determine a solution to the one dimensional heat equation with homogeneous Dirichlet boundary conditions, i.e.

$$\begin{aligned} \frac{\partial u}{\partial t} &= \alpha^2 \frac{\partial^2 u}{\partial x^2} \\ \text{BC} &: \quad u(0,t) = 0 = u(L,t) \\ \text{IC} &: \quad u(x,0) = f(x) \end{aligned}$$

   Evaluate the coefficients of the corresponding Fourier series for $f(x) = x$ and $L = 1$.Hence determine the solution of the above initial-boundary value problem.

   [40 marks]

2. Consider the second order differential equation:

$$Ly = 2x^2 y'' + xy' - (1+x)y = 0 \tag{1}$$

   (a) Classify the points $x \geq 0$ (do not include the point at infinity) as ordinary points, regular singular points, or irregular singular points.

   [5 marks]

   (b) Explain how you would classify the point at infinity (you need not carry out the calculations).

   [5 marks]

   (c) Explain how you would obtain a general solution of (1) about the point $x = 1$.

   [10 marks]

   (d) Use the appropriate series expansion about the point $x = 0$ to determine two independent solutions to (1). Determine the radius of convergence of one of these series.

   [40 marks]