Math 257/316, Midterm 1, Sections 101/102
19 October 2009

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.

Maximum score 80.

1. Consider the second order differential equation:

$$Ly = 6x^2y'' + xy' + (1+x)y = 0 \tag{1}$$

   (a) Classify the points $x \geq 0$ (including the point at ∞) as ordinary points, regular singular points, or irregular singular points.

   [10 marks]

   (b) If you were given $y(1) = 1$ and $y'(1) = 2$, what form of series expansion would you assume (you need not determine the expansion coefficients of this series)? What would be the minimal radius of convergence of this series?

   [5 marks]

   (c) Use the appropriate series expansion about the point $x = 0$ to determine two independent solutions to (1). You only need to determine the first three non-zero terms in each case.

   [35 marks]

2. Apply the method of separation of variables to determine the solution to the one dimensional heat equation with the following homogeneous Neumann boundary conditions:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi,\ t > 0 \\ \text{BC} &: \frac{\partial u}{\partial x}(0,t) = 0 = \frac{\partial u}{\partial x}(\pi,t) \\ \text{IC} &: u(x,0) = 1 \end{aligned}$$

[30 marks]