Math 257/316, Midterm 1, Section 101
20 October 2008

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.

Maximum score 100.

1. Let $f(x) = 1 + 2x$ on the interval $0 < x < 3$.

   (a) Sketch the even and odd periodic extensions of f with period 6.

   [5 marks]

   (b) Expand $f(x)$ in a half range Fourier cosine series. [15 marks]

   (c) Use the result in (b) to derive a series expansion for $\frac{\pi^2}{8}$.

   [10 marks]

   Hint: It may be useful to know that

$$\int_0^3 x\cos(\frac{n\pi x}{3})dx = \begin{cases} 0 \text{ if } n \text{ is even} \\ \frac{-18}{n^2\pi^2} \text{ if } n \text{ is odd} \end{cases}$$

2. Apply the method of separation of variables to determine the solution to the one dimensional heat equation with the following homogeneous boundary conditions (you need not give all the details of the different cases for the eigenvalue problem):

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \ t > 0 \\ \text{BC} &: \ u(0,t) = 0 = u(\pi,t) \\ \text{IC} &: \ u(x,0) = 2\cos x \sin x \end{aligned}$$

   [20 marks]

3. Consider the second order differential equation:

$$Ly = 2xy'' + (1+x)y' + y = 0 \tag{1}$$

   (a) Classify the points $x \geq 0$ (do not include the point at infinity) as ordinary points, regular singular points, or irregular singular points.

   [5 marks]

   (b) Use the appropriate series expansion about the point $x = 0$ to determine two independent solutions to (1). You only need to determine the first three non-zero terms in each case.

   [35 marks]