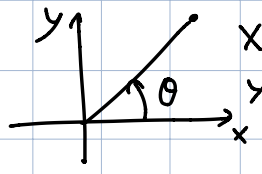
Lec 31 . Polar Coordinates & polar curves. §8.5

- slopes
- arc-length. § 8.6.
- Next lecture: Area, arc-length §8.6.

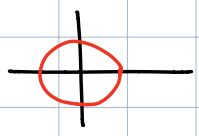


$x = r\cos\theta$
$y = r\sin\theta$

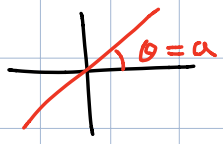
$r = \sqrt{x^2+y^2}$ $\tan(\theta) = \frac{y}{x}$

Polar curves

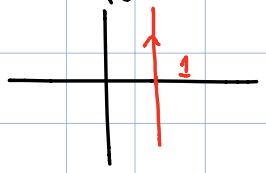
e.g. $r = a$ = const.



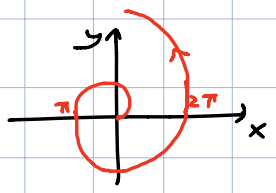
e.g. $\theta = a$ = const.



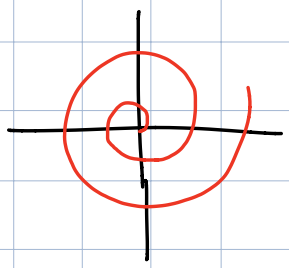
e.g. $r = \sec(\theta)$ ← $r = \frac{1}{\cos\theta}$ $r\cos\theta = 1$ $\therefore x = 1$



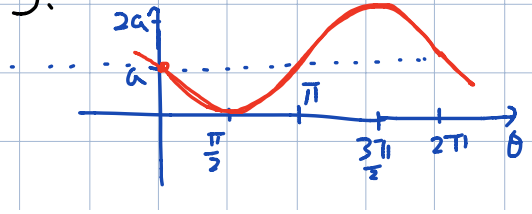
e.g. $r = \theta$:

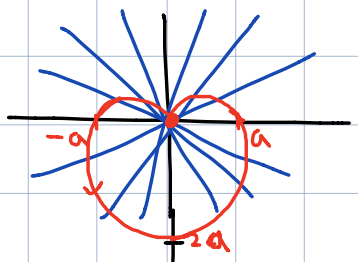


a spiral.



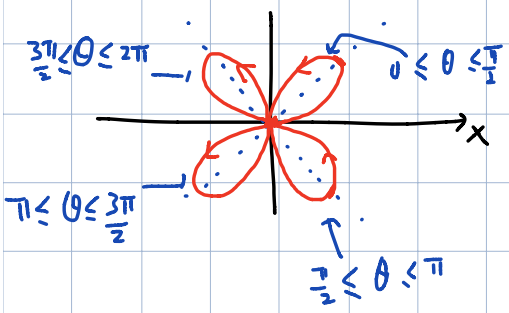
e.g. $a>0$, $r = a(1-\sin\theta)$

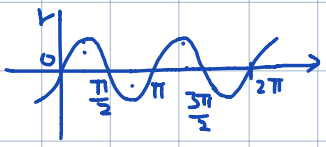




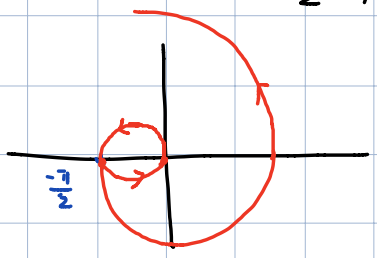
cardioid.

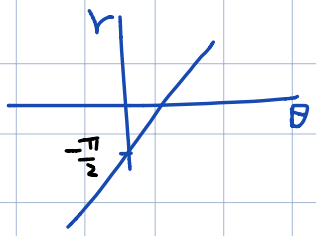
e.g. $r = \sin(2\theta)$





Ex $r = \theta - \frac{\pi}{2}$, $\theta \ge 0$





§. Slopes

Polar curve $r = f(\theta)$

$$x = r\cos\theta = f(\theta)\cos\theta$$

$$y = r\sin\theta = f(\theta)\sin\theta.$$

Ex In the cardioid $r = a(1 - \sin\theta)$ find those points where the tangent lines are horizontal or vertical:

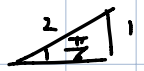
<sol>

Horizontal tangents $\Rightarrow$ $\frac{dy}{d\theta} = 0$

$$y = r\sin\theta = a(1-\sin\theta)\sin\theta$$

$$0 = \frac{dy}{d\theta} = a(\cos\theta - 2\sin\theta\cos\theta) = 1 \qquad \cos\theta(1 - 2\sin\theta) = 0$$

$\cos\theta = 0$ or $\sin\theta = \frac{1}{2}$



(In $0 \le \theta < 2\pi$): $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$. $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$.

· Vertical tangents: $\Rightarrow$

$$\frac{dx}{d\theta} = 0.$$

$$x = r\cos\theta = a(1-\sin\theta)\cos\theta$$

$$\frac{dx}{d\theta} = a(-\sin\theta - \cos^2\theta + \sin^2\theta)$$

$$= a(-\sin\theta + 2\sin^2\theta - 1)$$

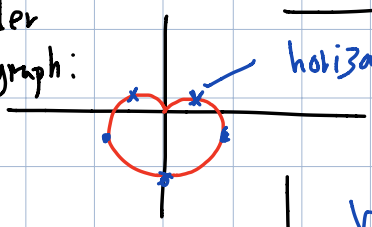
$$\frac{dx}{d\theta} = 0 \iff 2\sin^2\theta - \sin\theta - 1 = 0$$

$$\iff (2\sin\theta + 1)(\sin\theta - 1) = 0$$

$$\sin\theta = -\frac{1}{2}, \quad \sin\theta = 1$$

$$\therefore \quad \theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$$

Consider the graph:



Answer:

horizontal tangents at $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3}{2}\pi$

※ (not diff. at $\theta = \frac{\pi}{2}$)

vertical tangents: at $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$.