Math 121. Honours Integral Calculus.

Lec 1.

- Announcement:
  - Course webpage
  - exams: Midterms.
  - HW: { Written HW (WHW); Webwork: TBA.

Today: · motivation of integral, sums.
· Ch5. § 5.1 ~ 5.2

· Main theme of the course: "Integral calculus."

· $\int f(x)\,dx \xrightarrow{\frac{d}{dx}} f(x) \xrightarrow{\frac{d}{dx}} f'(x)$.

integral is an inverse operation of derivative.

Fundamental theorem of calculus.

$$f(b) - f(a) = \int_a^b \frac{d}{dx} f(x)\, dx.$$

* Can we these tools to compute area, volume, length, mass, etc.

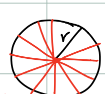
* application to · probability
  · differential eqns.

Motivating examples.

· dissection method

· dissect a shape into simple shapes and approximate.

EX. Circle of radius r.

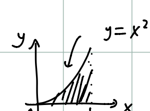


dissect to N simple pieces. For large N, $\theta = \frac{2\pi}{N}$. ← θ radian angle. approx. ≈ ∴ area $\approx \frac{1}{2}\theta r \cdot r$

∴ approx. area $= N \cdot \frac{1}{2}\theta r \cdot r = \cancel{N} \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\pi}{\cancel{N}} r \cdot r = \pi r^2$

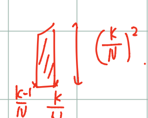
∴ area $= \pi r^2$ ▨

EX.

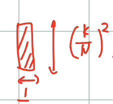


dissect.

$k = 1, 2, 3, \ldots, N$.



approx. ≈



↑ has area $\left(\frac{k}{N}\right)^2 \times \frac{1}{N}$

area $\underset{\text{approx.}}{\approx}$ $\left(\frac{1}{N}\right)^2 \times \frac{1}{N} + \left(\frac{2}{N}\right)^2 \times \frac{1}{N} + \left(\frac{3}{N}\right)^2 \times \frac{1}{N} + \cdots + \left(\frac{N-1}{N}\right)^2 \times \frac{1}{N} + \left(\frac{N}{N}\right)^2 \frac{1}{N}$

i.e.

area $\approx \frac{1}{N^3} \times \Big[\overbrace{1^2 + 2^2 + 3^2 + \cdots + (N-1)^2 + N^2}^{\text{formula} = \frac{N(N+1)(2N+1)}{6}} \Big] = \frac{1}{N^3} \times \frac{N(N+1)(2N+1)}{6} = \frac{1}{6} \cdot 1 \cdot \left[1 + \frac{1}{N}\right]\left[2 + \frac{1}{N}\right]$

To find the area, compute this and take $N \to \infty$.

$\xrightarrow{N \to \infty} \frac{1}{6} \times 2 = \underline{\underline{\frac{1}{3}}}$ ← the area. ▨

- Σ notation

$$a_m + a_{m+1} + a_{m+2} + \cdots + a_n = \sum_{j=m}^{n} a_j$$

ex $9 + 16 + 25 + 36 + 49 = 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = \sum_{j=3}^{7} j^2 = \sum_{j=0}^{4} (j+3)^2$

Basic properties $c, d \in \mathbb{R}$.

- linearity $\sum_{j=m}^{n} (c\,a_j + d\,b_j) = c \sum_{j=m}^{n} a_j + d \sum_{j=m}^{n} b_j$

proof
$$\sum_{j=m}^{n} [c\,a_j + d\,b_j] = \begin{array}{l} c\,a_m + d\,b_m \\ + c\,a_{m+1} + d\,b_{m+1} \\ \quad \vdots \\ + c\,a_n + d\,b_n \end{array} = \begin{array}{l} c\,[a_m + a_{m+1} + \cdots + a_n] \\ + d\,[b_m + b_{m+1} + \cdots + b_n] \end{array} = c \sum_{j=m}^{n} a_j + d \sum_{j=m}^{n} b_j$$ ▨