# A ONE PAGE SUMMARY OF RESEARCH

**YOUNG-HEON KIM**
UNIVERSITY OF TORONTO

My research interests lie at the intersection of analysis and geometry: spectral geometry and optimal transport theory.

Often one wants to determine the geometry from the spectrum of differential operators; for example, as Mark Kac once asked, "Can one hear the shape of a drum?" The relation between spectrum and geometry is not sharp in general, thus it is important to estimate the size of an *isospectral set* — the set of all the Riemannian metrics with the same spectrum on a given compact manifold. For compact manifolds with boundary, one of the best results in this direction has been that of Osgood, Phillips, and Sarnak, who showed a compactness result which implies the bounded plane domains having the same spectrum do not degenerate to singular shapes. I recently gave a significantly simpler treatment of their result and extended it to general flat surfaces with boundary, possibly with handles, such as a skillful person could make with paper, glue, and scissors (cutting off the conical points). A related study on the determinant of the Laplacian — a regularized value of the product of the infinitely many nonzero eigenvalues — is done in this work, extending some results of Wolpert, Bismut, and Bost.

Jointly with Robert McCann (University of Toronto), I have recently launched a research program in optimal transport theory, where one wants to understand optimizing phenomena occurring when transporting mass distributions in Economics, Physics, Probability, Analysis, Geometry, and recently in Biology. This profound subject started by Monge and Kantorovich has attracted a large community of researchers.

Regularity (smoothness) of optimal transportation is an important question for the same reasons that regularity of solutions to partial differential equations is crucial. For *general* cost functions governing the transportation, major progress has been achieved recently by Ma, Trudinger, & Wang and then by Loeper, following the pioneering works of Delanoë, Caffarelli, and Urbas for the *quadratic cost* $c(x, \bar{x}) = |x - \bar{x}|^2/2$. Together with McCann I recast their theory by introducing a new geometry with a pseudo-Riemannian metric $h$ induced by the transportation cost. This gives elementary proofs and extensions of some key ingredients in their works; a substantial simplification in Loeper's Hölder continuity theory; and new perspectives and research directions. In particular, a strengthened connection to global Riemannian geometry is emerging — especially in light of my counterexamples to regularity of optimal transportation on positively curved Riemannian manifolds extending the negative curvature counterexamples of Loeper, and also my work with McCann constructing a new class of examples obtained by Riemmanian submersions and products $S^{n_1} \times \cdots \times S^{n_k} \times \mathbf{R}^l$ where the Riemannian distance squared cost satisfies necessary conditions for regularity. Remarkably, an unexpected connection to symplectic geometry is also obtained: the graph of the optimal transportation map is a Lagrangian submanifold of the Kähler form of the pseudo-metric $h$.

For future research, one of my ambitions is to establish a substantially complete regularity theory of optimal transportation, which in particular extends Trudinger and Wang's local theory to the case of global manifold domains and more general costs.