Math 152, Spring 2008, Practise Test #2

• This test was given in 2006.
• Marks are indicated in the questions, out of a total of 20.

1. (a) (3 marks) Find all the solutions to the following system of linear equations:

\[
\begin{align*}
    x_1 + 2x_2 + x_3 &= 1 \\
    -x_1 + 2x_3 &= 0 \\
    -2x_1 - 2x_2 + x_3 &= -1
\end{align*}
\]

(b) (1) Can the vector \((1, 0, -1)\) be expressed as a linear combination of \((1, -1, -2)\), \((2, 0, -2)\) and \((1, 2, 1)\)? Give a reason.

(c) (2) Are the vectors \((1, -1, -2)\), \((2, 0, -2)\) and \((1, 2, 1)\) linearly independent? Give a reason.

2. \(A\) is a \(3 \times 3\) matrix. After row reductions, the following matrix results:

\[
U = \begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & 4 \\
0 & 0 & -1
\end{bmatrix}
\]

The row operations used were (in order): exchange row 1 and row 2; divide row 2 by 2; subtract row 1 from row 3

(a) (2) Find \(\det A\).

(b) (2) Is \(A\) invertible? Give a reason.

(c) (2) Find \(A\).

3. Consider the linear system

\[
\begin{bmatrix}
1 & 1 \\
1 & 2 \\
1 & 3
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
5 \\
6
\end{bmatrix}
\]

(a) (2) Show that this system has no solution.

(b) (2) Find the least squares solution.
4. Let \( T \) be the linear transformation acting on vectors in the plane \( \mathbb{R}^2 \) given by first rotating the vector by \( \pi/4 \) (45°) and then multiplying the resulting vector by the matrix

\[
\begin{bmatrix}
1 & 1 \\
0 & 1
\end{bmatrix}.
\]

Recall that \( \cos(\pi/4) = \sin(\pi/4) = 1/\sqrt{2} \).

(a) (2) What is the matrix for \( T \)?

(b) (2) What is the matrix for \( T^{-1} \)?