Very short answer questions

1. [2 marks] Each part is worth 1 mark.
   
   (a) Find $\int x^5 + 3x^3 \, dx$.
   
   $$\int x^5 + 3x^3 \, dx = \frac{x^6}{6} + \frac{3x^4}{4} + C$$

   (b) If $f(x)$ is a negative function on $[1, 2]$, is $\int_2^1 f(x) \, dx$ positive or negative?
   
   positive

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.
   
   (a) Write out the right Riemann sum for the function $f(x) = \sqrt{x}$ given by the regular partition of $[2, 5]$ into $n = 3$ subintervals. Your answer needs only to be ‘calculator ready’.

   \[
   \Delta x = 1
   \]

   Sum is $\left[ -\sqrt{3} + \sqrt{4} + \sqrt{5} \right]$

   (b) Use the fundamental theorem of calculus to differentiate $F(x) = \int_4^{4x} \sin(t) \, dt$

   \[
   u = 4x \quad F(u) = \int_4^u \sin(t) \, dt
   \]

   \[
   \frac{dF}{du} = \sin(u) \quad \text{by FTC}
   \]

   \[
   \frac{dF}{dx} = \frac{dF}{du} \cdot \frac{du}{dx} = \sin(u) \cdot 4 = 4 \sin(4x)
   \]
Long answer question — you must show your work

3. [4 marks] Using integration by substitution, evaluate \( \int_0^{\pi/3} \cos(x) \sin^3(x) \, dx \).

\[
\begin{align*}
\int_0^{\pi/3} \cos(x) \sin^3(x) \, dx &= \int_0^{\sqrt{3}/2} u^3 \, du \\
&= \left. \frac{u^4}{4} \right|_0^{\sqrt{3}/2} \\
&= \left( \frac{\sqrt{3}}{2} \right)^4 - 0^4 \\
&= \frac{9}{8} \left( \frac{\sqrt{3}}{2} \right)^4 - \frac{9}{32}
\end{align*}
\]