Very short answer questions

1. 2 marks Each part is worth 1 mark.
   
   (a) Find \( \int x^7 + 4x^2 \, dx \).
   
   \[
   = \frac{x^8}{8} + \frac{4x^3}{3} + C
   \]
   
   (b) If \( f(x) \) is a positive function on \([1, 2]\), is \( \int_2^1 f(x) \, dx \) positive or negative?
   
   negative

Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.
   
   (a) Write out the right Riemann sum for the function \( f(x) = \ln(x) \) given by the regular partition of \([1, 4]\) into \( n = 3 \) subintervals. Your answer needs only to be ‘calculator ready’.

   \[
   \text{partition} \quad 1 \quad 2 \quad 3 \quad 4 \quad \Delta x = 1
   \]

   \[
   \text{sum} = \ln(2) \cdot \Delta x + \ln(3) \cdot \Delta x + \ln(4) \cdot \Delta x
   \]

   \[
   = \ln(2) + \ln(3) + \ln(4) = \ln(24)
   \]

   (b) Use the fundamental theorem of calculus to differentiate \( F(x) = \int_5^{5x} \cos(t) \, dt \)

   \[
   u = 5x \quad F(u) = \int_5^u \cos(t) \, dt \quad \Rightarrow \quad \frac{dF}{du} = \cos(u) \ (\text{FTC})
   \]

   \[
   \frac{dF}{dx} = \frac{dF}{du} \cdot \frac{du}{dx} = \cos(u) \cdot 5 = 5 \cos(5x)
   \]
Long answer question — you must show your work

3. [4 marks] Using integration by substitution, evaluate \( \int_{0}^{\pi/4} \cos(x) \sin^3(x) \, dx \).

\[
\begin{align*}
  u &= \sin(x) \\
  du &= \cos(x) \, dx \\
  x &= 0 \quad u = \sin(0) = 0 \\
  x &= \frac{\pi}{4} \quad u = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}
\end{align*}
\]

\[
\int_{0}^{\pi/4} \frac{\cos(x) \sin^3(x) \, dx}{\sqrt{2}} = \int_{0}^{\frac{1}{\sqrt{2}}} u^3 \, du
\]

\[
= \left. \frac{u^4}{4} \right|_{0}^{\frac{1}{\sqrt{2}}}
\]

\[
= \frac{\left(\frac{1}{\sqrt{2}}\right)^4}{4} - \frac{0^4}{4}
\]

\[
= \frac{1}{4} - \frac{1}{16}
\]

\[
= \frac{1}{4} \left[ \frac{1}{16} \right] = \frac{1}{16}
\]