

Mathematics 101 — Midterm — 45 minutes

13 & 14 February 2020

- The test consists of 12 pages and 6 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is split into 3 dependent sub-questions. Question 5 is a single question. Question 6 is split into 2 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.
- No memory aids. No calculators. No communication devices or other electronic devices.
- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

Student number								
Section								
Preferred Name								
Given Name								
Family Name								

Question:	1	2	3	4	5	6	Total
Points:	8	12	8	8	4	4	44
Score:							

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Riemann Sum and FTC

1. 8 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the infinite sum

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{6i^2}{n^3 \left(\frac{i^3}{n^3} + 1 \right)}$$

by first writing it as a definite integral and then evaluating it.

Answer:

- (b) Define $F(x)$ and $g(x)$ by $F(x) = \int_{2\sqrt{\pi}}^x t \cos(t^2) dt$ and $g(x) = x^3 F(x)$. Calculate $g'(\sqrt{\pi})$.

Answer:

Indefinite Integrals

2. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the indefinite integral $\int 4(x - 2)^7 \sin((x - 2)^4) dx$.

Answer:

(b) Calculate the indefinite integral $\int (2 + 3 \sin \theta)^{\frac{9}{2}} \cos \theta d\theta$.

Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int 2x^5\sqrt{1-x^2} dx$ for $|x| \leq 1$.

Answer:

Definite Integrals

3. 8 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $\int_{\pi/4}^{\pi/3} \frac{\sec^2(x)}{\tan^{3/2}(x)} dx$.

Answer:

(b) Calculate $\int_0^1 \arctan(4x) dx$.

Answer:

Areas, volumes and work

Please write your answers in the boxes. **Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.**

4. (a) Sketch by hand the finite area enclosed between the curves defined by the functions $y^2 - 3 = -x$ and $x = -1$

Answer:

- (b) Write the definite integral with specific limits of integration that determines this finite area.

Answer:

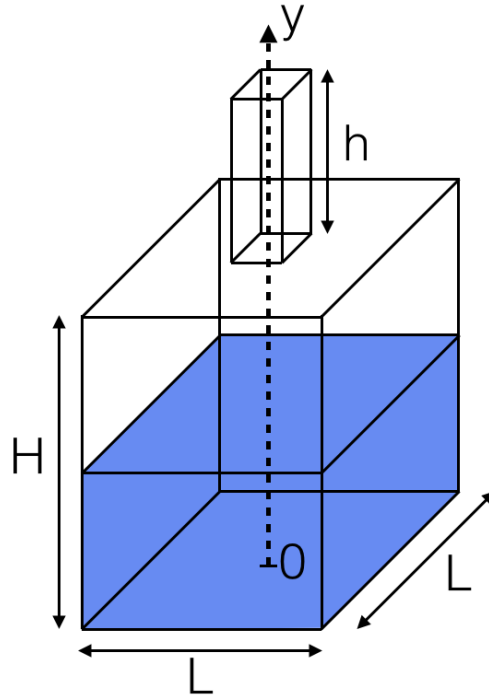
(c) 2 marks Evaluate the integral.

Answer:

5. 4 marks Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $y = 5\sqrt{x} - 2$ and $y = x + 2$ about the vertical line $x = -3$. **Do not evaluate the integral.**

Answer:

6. A tank of height H and of square cross section of edge length L is half full with water of density $\rho = 1000\text{kg/m}^3$. The top of the tank features a spout of height h . We take the vertical axis y upwards oriented with its origin at the bottom of the tank. We assume gravity acceleration is $g = 10\text{m/s}^2$. We take $H = 4\text{m}$, $L = 6\text{m}$ and $h = 7\text{m}$.



- (a) 2 marks Formulate the total work to pump the water out of the tank by the top of the spout as a definite integral.

Answer:

(b) 2 marks Evaluate the definite integral.

Answer:

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