Mathematics 101 — Midterm — 45 minutes

13 & 14 February 2020

- The test consists of 12 pages and 6 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is split into 3 dependent sub-questions. Question 5 is a single question. Question 6 is split into 2 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.

- No memory aids. No calculators. No communication devices or other electronic devices.

- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

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Riemann Sum and FTC

1. 8 marks Each part is worth 4 marks. Please write your answers in the boxes.
   (a) Calculate the infinite sum

   \[
   \lim_{n \to \infty} \sum_{i=1}^{n} \frac{3i^2}{n^3} \sqrt{\frac{i^3}{n^3} + 2}
   \]

   by first writing it as a definite integral and then evaluating it.

   Answer:

(b) Define \( F(x) \) and \( g(x) \) by \( F(x) = \int_{0}^{x^2} e^{-t^2} \, dt \) and \( g(x) = F(\sin x) \). Calculate \( g'(\pi/4) \).

   Answer:
Indefinite Integrals

2. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the indefinite integral \( \int 3(x + 1)^5 \sin ((x + 1)^3) \, dx \).

Answer:

(b) Calculate the indefinite integral \( \int (1 + 3 \sin \theta)^{11/2} \cos \theta \, d\theta \).

Answer:
(c) (A Little Harder): Calculate the indefinite integral \( \int \frac{\ln(9+x^2)}{x^2} \, dx \).

Answer:
Definite Integrals

3. [8 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate \( \int_{-\pi/2}^{\pi/2} (3 + x^3) \cos(x) \, dx \).

Answer:

(b) Calculate \( \int_{0}^{1} \frac{3x^2}{2x^2 + 2} \, dx \).

Answer:
Areas, volumes and work

Please write your answers in the boxes. Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.

4. (a) [2 marks] Sketch by hand the finite area enclosed between the curves defined by the functions $y = 1 - x^2$ and $2y + 2 = 2x$

Answer:

(b) [4 marks] Write the definite integral with specific limits of integration that determines this finite area.

Answer:
(c) [2 marks] Evaluate the integral.

Answer:
5. [4 marks] Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $y = 4\sqrt{x} - 2$ and $y = x+1$ about the vertical line $x = -1$. Do not evaluate the integral.

Answer:
6. A tank of height $H$ and of square cross section of edge length $L$ is half full with water of density $\rho = 1000\, kg/m^3$. The top of the tank features a spout of height $h$. We take the vertical axis $y$ upwards oriented with its origin at the bottom of the tank. We assume gravity acceleration is $g = 10\, m/s^2$.
We take $H = 4\, m$, $L = 4\, m$ and $h = 2\, m$.

(a) 2 marks Formulate the total work to pump the water out of the tank by the top of the spout as a definite integral.

Answer:
(b) 2 marks Evaluate the definite integral.

Answer:
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