Mathematics 101 — Midterm — 45 minutes

14 & 15 February 2019

- The test consists of 10 pages and 5 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is a single question. Question 5 is split into 3 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.

- No memory aids. No calculators. No communication devices or other electronic devices.

- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

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Score:
Indefinite Integrals

1. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the indefinite integral \( \int \arctan \left( \frac{1}{x} \right) \, dx \) for \( x > 0 \).

Answer:

(b) Calculate the indefinite integral \( \int -3x \sqrt{3 + 3x} \, dx \) for \( x < 1 \).

Answer:
(c) (A Little Harder): Calculate the indefinite integral \( \int \frac{x^2 + x + 4}{x^3 + 3x + 2x^2 + 3} \, dx \).

Answer:
Definite Integrals

2. [8 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate \( \int_{1}^{e} \frac{1-\ln(x)}{x} \, dx \).

Answer:

(b) Calculate \( \int_{1}^{5} \frac{x-4}{\sqrt{8x-14-x^2}} \, dx \).

Answer:
Riemann Sum and FTC

3. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Which definite integral corresponds to \( \lim_{n \to \infty} \sum_{i=1}^{n} \ln\left(\frac{3i}{n} - \frac{3}{n} + 1\right) \sin\left(\frac{6i}{n} - \frac{6}{n}\right) \frac{3}{n} \)?
   
   \( \begin{align*}
   (A) & \quad \int_{0}^{3} \ln(x + 1) \sin(2x) \, dx \\
   (B) & \quad 3 \int_{0}^{1} \ln(x + 1) \sin(2x) \, dx \\
   (C) & \quad \frac{1}{2} \int_{0}^{6} \ln(x + 1) \sin(2x) \, dx \\
   (D) & \quad \int_{0}^{6} \ln\left(\frac{x}{2} + 1\right) \sin(x) \, dx \\
   (E) & \quad 2 \int_{0}^{3} \ln\left(\frac{x}{2} + 1\right) \sin(x) \, dx \\
   \end{align*} \)

Answer:

(b) Define \( F(x) \) and \( g(x) \) by \( F(x) = \int_{-1}^{x} t^2 \, dt \) and \( g(x) = (F(x^2))^4 \). Calculate \( g'(1) \).

Answer:
(c) Let \( F(x) = \int_{x^2}^{x^3} 9e^{t^2} \, dt \). Find the equation of the tangent line to the graph of \( y = F(x) \) at \( x = 1 \). Tip: recall that the tangent line to the graph of \( y = F(x) \) at \( x = x_0 \) is given by the equation

\[
y = F(x_0) + F'(x_0)(x - x_0).
\]

Answer:
Areas and volumes

Please write your answers in the boxes. Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.

4. [4 marks] Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $y = \sqrt{x - 1}$ and $x = 1 + \sqrt{y}$ about the horizontal line $y = -2$. Do not evaluate the integral.

Answer:
5. (a) 2 marks Sketch by hand the finite area enclosed by \( y^2 + x = 0 \) and \( 2y - x = 15 \)

Answer:

(b) 4 marks Write a definite integral with specific limits of integration that determines this finite area.

Answer:
(c) [2 marks] Evaluate the integral to compute the area enclosed.

Answer: