Mathematics 101 — Midterm — 45 minutes

14 & 15 February 2019

- The test consists of 10 pages and 5 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is a single question. Question 5 is split into 3 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.
- No memory aids. No calculators. No communication devices or other electronic devices.
- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

Student number								
Section								
Preferred Name								
Given Name								
Family Name								

Question:	1	2	3	4	5	Total
Points:	12	8	12	4	8	44
Score:						

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Indefinite Integrals

1. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

   (a) Calculate the indefinite integral $\int \cos x \ln(\sin x)\, dx$ for $\sin x > 0$.

   Answer:

   (b) Calculate the indefinite integral $\int 2x\sqrt{3-2x}\, dx$ for $x < 3/2$.

   Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int \frac{\sqrt{x^2-16}}{x^2}\,dx, x > 4$. Use the following known result: $\int \sec x\,dx = \ln|\sec x + \tan x| + C$. **Write your final answer without any trigonometric function**.

Answer:

Definite Integrals

2. [8 marks] Each part is worth 4 marks. Please write your answers in the boxes.

   (a) Calculate $\int_1^5 \frac{x-1}{x^2(x+1)}\,dx$.

   Answer:

   (b) Calculate $\int_2^3 \frac{x-2}{\sqrt{4x-2-x^2}}\,dx$.

   Answer:

Riemann Sum and FTC

3. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

   (a) Which definite integral corresponds to $\lim_{n\to\infty}\sum_{i=1}^{n}(\frac{6i}{n}+e^{9\frac{i^2}{n^2}})\sin(\frac{2i}{n}+1)\frac{1}{n}$?

   (A) $\int_0^9(x+e^{x^2})\sin(x+1)dx$

   (B) $\int_0^6(x+e^{\frac{1}{4}x^2})\sin(\frac{1}{3}x+1)dx$

   (C) $\int_0^3(2x+e^{x^2})\sin(\frac{2}{3}x+1)dx$

   (D) $\int_0^2(3x+e^{\frac{9}{4}x^2})\sin(x+1)dx$

   (E) $\int_0^1(6x+e^{9x^2})\sin(2x+1)dx$

   Answer:

   (b) Define $F(x)$ and $g(x)$ by $F(x)=\int_1^x \ln t\,dt$ and $g(x)=(F(x^2))^2$ for $x>1$. Calculate $g'(2)$. Give the answer as a function of $\ln 2$.

   Answer:

(c) Let $F(x) = \int_{x^2}^{x^3} 2e^{t^2}\, dt$. Find the equation of the tangent line to the graph of $y = F(x)$ at $x = 1$. Tip: recall that the tangent line to the graph of $y = F(x)$ at $x = x_0$ is given by the equation $y = F(x_0) + F'(x_0)(x - x_0)$.

Answer:

Areas and volumes

Please write your answers in the boxes. **Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.**

4. [4 marks] Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $x = 10 - (y-1)^2$ and $x = 2 + (y-1)^2$ about the vertical line $x = 1$. **Do not evaluate the integral.**

Answer:

5. (a) [2 marks] Sketch by hand the finite area enclosed by $y^2 + 1 = -x$ and $2y = -9 - x$

Answer:

(b) [4 marks] Write a definite integral with specific limits of integration that determines this finite area.

Answer:

(c) [2 marks] Evaluate the integral to compute the area enclosed.

Answer: