Mathematics 101 — Midterm — 45 minutes

14 & 15 February 2019

• The test consists of 10 pages and 5 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is a single question. Question 5 is split into 3 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.

• No memory aids. No calculators. No communication devices or other electronic devices.

• Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

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This page has been left blank for your workings.
Indefinite Integrals

1. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the indefinite integral \( \int \frac{\ln x}{\sqrt{x}} \, dx \) for \( x > 0 \).

Answer:

(b) Calculate the indefinite integral \( \int -2x\sqrt{3 + 2x} \, dx \) for \( x > -3/2 \).

Answer:
(c) (A Little Harder): Calculate the indefinite integral $\int \frac{x^2 + x + 3}{x^3 + 4x - x^2 - 4} \, dx$. 

Answer:
Definite Integrals

2. [8 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate \( \int_{-\pi/2}^{\pi/2} 3 \cos^3 x \, dx \).

\[
\text{Answer:} \\
\]

(b) Calculate \( \int_{-2}^{-1} \frac{x+2}{\sqrt{-4x^2-2x}} \, dx \).

\[
\text{Answer:} \\
\]
Riemann Sum and FTC

3. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Which definite integral corresponds to \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{\sqrt{i^2+9n^2}}{i^2} \)?

(A) \( \int_{0}^{3} \frac{\sqrt{x^2+1}}{x^2} \, dx \)
(B) \( 3 \int_{0}^{1} \frac{\sqrt{x^2+1}}{x^2} \, dx \)
(C) \( \frac{1}{3} \int_{0}^{1} \frac{\sqrt{x^2+1}}{x^2} \, dx \)
(D) \( \int_{0}^{1} \frac{\sqrt{x^2+9}}{x^2} \, dx \)
(E) \( \int_{0}^{3} \frac{\sqrt{x^2+9}}{x^2} \, dx \)

Answer:

(b) Define \( F(x) \) and \( g(x) \) by \( F(x) = \int_{2x}^{x} \cos^2 t \, dt \) and \( g(x) = x \, F(x) \). Calculate \( g'(\pi) \).

Answer:
(c) Let \( F(x) = \int_{x^3}^{x^2} 7e^{t^2} \, dt \). Find the equation of the tangent line to the graph of \( y = F(x) \) at \( x = 1 \). Tip: recall that the tangent line to the graph of \( y = F(x) \) at \( x = x_0 \) is given by the equation \( y = F(x_0) + F'(x_0)(x - x_0) \).

Answer:
Areas and volumes

Please write your answers in the boxes. **Do not use absolute values in your expressions, always work out**: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.

4. [4 marks] Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between \( x = -(y - 4)^2 \) and \( x = -2 - y \) about the vertical line \( x = 1 \). **Do not evaluate the integral.**

\[
\text{Answer:}
\]

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5. (a) 2 marks Sketch by hand the finite area enclosed by \( y^2 = 3 - x \) and \( 3y = x + 1 \)

Answer:

(b) 4 marks Write a definite integral with specific limits of integration that determines this finite area.

Answer:
(c) 2 marks Evaluate the integral to compute the area enclosed.

Answer: