Mathematics 101 — Midterm — 45 minutes

14 & 15 February 2019

- The test consists of 10 pages and 5 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is a single question. Question 5 is split into 3 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.

- No memory aids. No calculators. No communication devices or other electronic devices.

- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

---

<table>
<thead>
<tr>
<th>Student number</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Preferred Name</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Given Name</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Family Name</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>Question:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points:</td>
<td>12</td>
<td>8</td>
<td>12</td>
<td>4</td>
<td>8</td>
<td>44</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Score:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>
This page has been left blank for your workings.
Indefinite Integrals

1. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.
   
   (a) Calculate the indefinite integral \( \int x^2 \sin x \, dx \) for \( x > 0 \).

   Answer:

   

   (b) Calculate the indefinite integral \( \int 4x \sqrt{3 - 4x} \, dx \) for \( x < 3/4 \).

   Answer:
(c) (A Little Harder): Calculate the indefinite integral $\int \frac{\sqrt{x^2-9}}{x^2} \, dx$, $x > 3$.

Use the following known result: $\int \sec x \, dx = \ln |\sec x + \tan x| + C$. Write your final answer without any trigonometric function.

Answer:
Definite Integrals

2. [8 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate \( \int_0^{\pi/3} \sec^{3/2} x \tan x \, dx \).

Answer:

(b) Calculate \( \int_{-1}^{0} \frac{x+1}{\sqrt{2x+1}-x} \, dx \).

Answer:
Riemann Sum and FTC

3. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Which definite integral corresponds to \( \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{i}{n} + 1 \right) e^{-2 \left( \frac{i}{n} \right)^2} \)?

(A) \( 2 \int_{0}^{1} xe^{-2(x-1)^2} \, dx \)
(B) \( 2 \int_{0}^{1} (x + 1)e^{-2x^2} \, dx \)
(C) \( \int_{1}^{2} xe^{-2(x-1)^2} \, dx \)
(D) \( \int_{1}^{2} (x + 1)e^{-2x^2} \, dx \)
(E) \( \int_{0}^{1} (x + 1)e^{-2x^2} \, dx \)

Answer:

(b) Define \( F(x) \) and \( g(x) \) by \( F(x) = \int_{0}^{x} \sin^2 t \, dt \) and \( g(x) = x \, F(x^3) \). Calculate \( g'(\pi^{1/3}) \).

Answer:
(c) Let \( F(x) = \int_{x^2}^{x^3} 3e^{t^2} \, dt \). Find the equation of the tangent line to the graph of \( y = F(x) \) at \( x = 1 \). Tip: recall that the tangent line to the graph of \( y = F(x) \) at \( x = x_0 \) is given by the equation \( y = F(x_0) + F'(x_0)(x - x_0) \).

Answer:
Areas and volumes

Please write your answers in the boxes. Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.

4. [4 marks] Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $y = (x - 2)^2$ and $y = x + 4$ about the horizontal line $y = 10$. Do not evaluate the integral.

Answer:
5. (a) 2 marks Sketch by hand the finite area enclosed by \( y^2 - x = 0 \) and \( x - 3y = 10 \)

Answer:

(b) 4 marks Write a definite integral with specific limits of integration that determines this finite area.

Answer:
(c) 2 marks Evaluate the integral to compute the area enclosed.

Answer: