1 Initial meeting

We gave an introduction to the course. We discussed the topics. It seems a good idea to include boundary layer. I sent a request for a classroom for Mon Wed 10am-11:15am.

2 Lebesgue integral

1. Motivation: weak solutions are limits of $L^2$ functions, and we need limit properties in $L^2$. Lebesgue integral sets the framework for $L^p$.
2. review Riemann integral.
3. rough definition for Lebesgue integral.
4. Lebesgue measure, measurable sets and measurable functions.
5. basic properties of Lebesgue integral.

6. Three theorems on the limits of Lebesgue integrals of sequences of functions
   (a) Monotone convergence theorem
   (b) Fatou Lemma
   (c) Dominated convergence theorem
7. $L^p$ spaces, Hölder inequality and Cauchy-Schwartz inequality

References: This is usually taught in a real analysis course, e.g., MATH 507 with textbook [3] by Folland. I learned the material from [10] by Wheeden and Zygmund. See Wikipedia for a good summary.

3 Weak derivatives and Sobolev spaces

1. weak derivatives: motivation, definition and examples

2. Sobolev spaces: definition and examples
3. Sobolev inequalities and imbedding, best representative
4. dimension analysis and scaling
5. Gagliardo-Nirenberg inequality

References: Chapter 5 of Evans [2], or Chapter 7 of Gilbarg-Trudinger [5].
4 Weak limits and compactness

1. strong and weak limits.
2. oscillation, concentration, and escaping to infinity
3. weak compactness
4. compactness of Sobolev imbedding

References: Chapter 5 of Evans [2], or Chapter 7 of Gilbarg-Trudinger [5].

5 heat equation

1. fundamental solution and Green function, semigroup form.
2. Galerkin method: approximation solutions in finite dimensional subspaces
3. local existence and a priori bounds of approximation solutions for the heat equation
4. more on the a priori bound
5. weak limit of the functions
6. weak limit of the equation
7. time-independent a priori bound
8. the semigroup method

References: §7.1 and §7.4 of Evans [2].

6 An introduction to incompressible fluid flows

1. Incompressible Euler and Navier-Stokes equations
2. zero normal velocity, no-slip, and Navier boundary conditions
3. Derivation of these equations from physical principles: assumptions, balance of mass and momentum, stress tensor for Newtonian fluid, equation of state

References: Chorin and Marsden [1, §1.1] and Tsai [9, §1.1-1.2].
7 A few fluid references

1. Ladyzhenskaya [6]: classical book for PDE theory of NS
2. Temam [8]: good for both theoretical and numerical analysis (finite element)
4. Chorin and Marsden [1]: We will follow it for the derivation of Euler and NS.
5. Galdi [4]: comprehensive treatise on stationary Navier-Stokes equations
6. Tsai [9]

References


