MATH 515 summary
Tai-Peng Tsai

1 Initial meeting
We gave an introduction to the course. We discussed the topics. It seems a good idea to include boundary layer. I sent a request for a classroom for Mon Wed 10am-11:15am.

2 Lebesgue integral
1. Motivation: weak solutions are limits of $L^2$ functions, and we need limit properties in $L^2$. Lebesgue integral sets the framework for $L^p$.
2. review Riemann integral.
3. rough definition for Lebesgue integral.
4. Lebesgue measure, measurable sets and measurable functions.
5. basic properties of Lebesgue integral.
6. Three theorems on the limits of Lebesgue integrals of sequences of functions
   (a) Monotone convergence theorem
   (b) Fatou Lemma
   (c) Dominated convergence theorem
7. $L^p$ spaces, Hölder inequality and Cauchy-Schwartz inequality

References: This is usually taught in a real analysis course, e.g., MATH 507 with textbook [2] by Folland. I learned the material from [8] by Wheeden and Zygmund. See Wikipedia for a good summary.

3 Weak derivatives and Sobolev spaces
1. weak derivatives: motivation, definition and examples
2. (Sobolev spaces, etc)
4 A few fluid references

1. Ladyzhenskaya [4]: classical book for PDE theory of NS
2. Temam [6]: good for both theoretical and numerical analysis (finite element)
4. Chorin and Marsden [1]: We will follow it for the derivation of Euler and NS.
5. Galdi [3]: comprehensive treatise on stationary Navier-Stokes equations
6. Tsai [7]

References


