1. The vector field \( \vec{F} = \frac{2z\hat{j} - 2y\hat{k}}{x^2 + y^2 + z^2} \) has a vector potential. Check its screening test, and find a vector potential \( \vec{A} \).

2. Let \( A = \frac{1}{2} \begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix} \) and \( \omega = (a, b, c)^T \) for constants \( a, b, c \). Verify that
\[
A\vec{x} = \frac{1}{2} \omega \times \vec{x}, \quad \text{curl}(A\vec{x}) = \omega.
\]

Here we treat both \( \vec{x} = (x, y, z)^T \) and \( \omega \) as column vectors.

3. The vector field \( \vec{F}(x, y) = (-y, x)^T \) at the point \((2, 1)\) can be decomposed by Taylor expansion in the form
\[
\vec{F}(x, y) = \vec{F}_0 + S \left( \frac{x - 2}{y - 1} \right) + A \left( \frac{x - 2}{y - 1} \right) + \text{error}
\]
where \( \vec{F}_0 \) is a constant vector, \( S \) is a constant symmetric matrix, \( A \) is a constant anti-symmetric matrix, and “error” means a vector function \( \vec{G}(x, y) \) satisfying
\[
\lim_{(x, y) \to (2, 1)} \frac{|\vec{G}(x, y)|}{|x - 2| + |y - 1|} = 0.
\]
Find \( \vec{F}_0, S \) and \( A \). Is there any local stretching? Is there any local rotation?

4. Repeat the previous problem for \( \vec{F}(x, y) = (x - y, x + y)^T \).

5. Repeat the previous problem for \( \vec{F}(x, y) = \frac{(-y, x)^T}{x^2 + y^2} \).

6. Compute the outward flux \( \iint_S \vec{F} \cdot d\vec{S} \) where \( \vec{F} = \frac{2z\hat{j} - 2y\hat{k}}{x^2 + y^2 + z^2} \) and \( S \) is the boundary of the solid \( V \) enclosed by \( z = (x - 3)^2 + y^2 - 4 \) and \( z = \cos(x + y) + 10 \).

7. Redo H7 Problem 1(b) using Divergence theorem: Let \( S \) be the upper hemisphere \( x^2 + y^2 + z^2 = 4, \ z \geq 0 \). Find the upward flux \( \iint_S \vec{F} \cdot d\vec{S} \) for \( \vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k} \).