1. Let $S$ be the upper hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$.
   (a) Find the mass of $S$ if it has density $\rho = z$.
   (b) Find the upward flux $\iint_S \vec{F} \cdot d\vec{S}$ for $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$.

2. Let $S$ be the part of the paraboloid $z = x^2 + y^2$ that is below the plane $2x + z = 8$.
   (a) Find the mass of $S$ if it has density $\rho = z$.
   (b) Find the downward flux $\iint_S \vec{F} \cdot d\vec{S}$ for $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$.
   For this problem you only need to setup the integrals with explicit limits. Do not evaluate the integrals.

3. (You can skip this problem since it is difficult.)
   The Möbius strip can be obtained as follows. Let $C$ be the circle $x^2 + y^2 = 1$ in the $xy$-plane. A line segment $L$ of length 1 is rotated around its midpoint $P$ at the same time as $P$ moves around the circle $C$, in such a way that as $P$ moves once around $C$, $L$ makes a half-turn about $P$. If $L$ is initially placed parallel to the $z$-axis with $P$ at $(1,0,0)$, the Möbius strip $M$ can be parametrized as
   \[
   \vec{r}(t, \theta) = \left( (1 - t \sin \frac{\theta}{2}) \cos \theta, (1 - t \sin \frac{\theta}{2}) \sin \theta, t \cos \frac{\theta}{2} \right), \quad -\frac{1}{2} \leq t \leq \frac{1}{2}, \quad 0 \leq \theta \leq 2\pi.
   \]
   Hence $\theta$ is the angle of $P = \vec{r}(0, \theta) = (\cos \theta, \sin \theta, 0)$ around the $z$-axis, and $t$ is the distance of a point on $L$ to $P$. Is $\vec{N} = \vec{r}_t \times \vec{r}_\theta$ a normal vector field on $M$? Explain why this does not imply that $M$ is orientable.

4. Compute the divergence and curl of the vector field $\vec{F} = (x^2y, y^2z, z^2x)$.

5. Does the vector field $\vec{F}$ in the previous problem has a vector potential? Explain why.

6. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. Verify the following identities:
   (a) $\nabla r = \vec{r}/r$, \hspace{1cm} (b) $\nabla \times \vec{r} = 0$, \hspace{1cm} (c) $\nabla (1/r) = -\vec{r}/r^3$, \hspace{1cm} (d) $\nabla \ln r = \vec{r}/r^2$
   Note. This is a problem from the practice homework with a solution. It is so important that I copy it here, and you should do it by yourself.

7. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. Find the formulas for $\nabla r^m$ and $\Delta r^m$ for $m \in \mathbb{R}$, $m \neq 0$.

8. The vector field $\vec{F} = x\hat{i} + 2y\hat{j} + (x - 3z)\hat{k}$ has a vector potential. Check its screening test, and find a vector potential $\vec{A}$ with $A_3 = 0$, and another vector potential $\vec{B}$ with $B_1 = 0$. 