1. Let $\vec{F}$ be a continuously differentiable vector field in an open connected subset $D$ of $\mathbb{R}^3$. Determine whether the following statements are true or false, and explain why.

(a) If $\text{curl} \vec{F} = 0$, then $\vec{F}$ is conservative in $D$.

(b) If $\text{curl} \vec{F} = 0$, then the line integral $\int_C \vec{F} \cdot d\vec{r}$ vanishes for any closed curve $C$ in $D$.

(c) If the line integral $\int_C \vec{F} \cdot d\vec{r}$ vanishes for any closed curve $C$ in $D$, then $\text{curl} \vec{F} = 0$.

(d) If $\vec{F}$ is not conservative in $D$, then there is a closed curve $C$ in $D$ such that the line integral $\int_C \vec{F} \cdot d\vec{r}$ is nonzero.

2. Let $\vec{F}(x,y) = \frac{y^3}{(x^2 + y^2)^2} \hat{i} - \frac{xy^2}{(x^2 + y^2)^2} \hat{j}$ for $(x,y)$ in $D = \mathbb{R}^2 \setminus \{(0,0)\}$.

(a) Compute $\partial_x F_2 - \partial_y F_1$.

(b) Find $I_1 = \int_{C_1} \vec{F} \cdot d\vec{r}$ where $C_1$ is the circle $x^2 + y^2 = 4$ in the $xy$-plane, oriented counterclockwise.

(c) Find $I_2 = \int_{C_2} \vec{F} \cdot d\vec{r}$ where $C_2$ is the circle $(x+4)^2 + y^2 = 4$ in the $xy$-plane, oriented counterclockwise.

3. Let $\vec{H} = 2\vec{F} + \vec{G}$, where $\vec{F}$ is as in the previous problem, and $\vec{G}(x,y) = -\frac{y}{x^2 + y^2} \hat{i} + \frac{x}{x^2 + y^2} \hat{j}$. Show that $\vec{H}$ is conservative in $D$ by finding its potential $\phi$ explicitly.

Hint. $\int \frac{dx}{(x^2 + a^2)^2} = \frac{1}{2a^2} \frac{x}{x^2 + a^2} + \frac{1}{2a^2} \int \frac{dx}{x^2 + a^2}$, $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a}$.

4. Find a parametric representation of the plane through the point $(3, 1, 7)$ that are parallel to the vectors $\hat{j} - \hat{k}$ and $\hat{k} - 2\hat{i}$. You must specify the domain of your parameters.

5. Find a parametric representation for the hyperboloid $z^2 = x^2 + y^2 + 1$, $z > 0$. You must specify the domain of your parameters.

6. Find a parametric representation for the hyperboloid $z^2 = x^2 + y^2 - 1$. You must specify the domain of your parameters.

7. Find a parametric representation for the part of the sphere $x^2 + y^2 + z^2 = 4$ that satisfies $x \geq 1$. You must specify the domain of your parameters.

8. Find a parametric representation for the part of the sphere $x^2 + y^2 + z^2 = 4$ that satisfies $x \leq 1$. You must specify the domain of your parameters.

9. Sketch the surface $\vec{F}(u,v) = (u, v^2, v)$, $-2 \leq u \leq 2$, $-2 \leq v \leq 2$. 

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