1. The function $f(x) = x^3 - x$ has local minimum $f\left(\frac{1}{\sqrt{3}}\right) = -\frac{2\sqrt{3}}{9}$. Find the centres of the osculating circles of the curve $y = x^3 - x$ at the points $P\left(\frac{1}{\sqrt{3}}, -\frac{2\sqrt{3}}{9}\right)$ and $Q(1, 0)$.

2. Find the unit tangent, unit normal and binormal vectors and the curvature and torsion of the curve
$$\vec{r}(t) = \sin(3t) \hat{i} + \cos(3t) \hat{j} + 4t \hat{k}.$$ 

3. Find the unit tangent, unit normal and binormal vectors and the curvature and torsion of the curve
$$\vec{r}(t) = t^2 \hat{i} + 2t \hat{j} + \frac{1}{3} t^3 \hat{k}.$$ 

4. Suppose that the curve $C$ is the intersection of the cylinder $x^2 + y^2 = 1$ with the surface $z = x^2 - y^2$.

(a) Find a parameterization of $C$.

(b) Determine the curvature of $C$ at the point $P = (1/\sqrt{2}, 1/\sqrt{2}, 0)$.

(c) Find the osculating plane to $C$ at the point $P$. In general, the osculating plane to a curve $\vec{r}(t)$ at the point $\vec{r}(t_0)$ is the plane which fits the curve best at $\vec{r}(t_0)$. It passes through $\vec{r}(t_0)$ and has normal vector $\hat{B}(t_0)$.

(d) Find the radius and the centre of the osculating circle to $C$ at the point $P$.

5. Find the mass and centre of mass of the curve
$$\vec{r}(t) = t^2 \hat{i} + 2t \hat{j} + \frac{1}{3} t^3 \hat{k}, \quad 0 \leq t \leq 2,$$
with density $\rho(t) = t^2$.

6. Suppose Mr. Hinton hit a baseball 1 m above the ground toward the centre field fence, which is 3 m higher than and 120 m from the home plate. Suppose the ball leaves the bat with 40 m/s speed at angle 50° above the horizontal. Is it a home run?

Hint. $\cos 50° \approx 0.6428$, $\sin 50° \approx 0.7660$, gravity constant $g = 9.8 \text{ m/s}^2$.

7. Sketch each of the following vector fields, by drawing a figure like Figure 2.1.1 in the CLP-IV text.

(a) $\vec{V}(x, y) = 2x \hat{i} - \hat{j}$.

(b) $\vec{V}(x, y) = \frac{y \hat{i} - x \hat{j}}{\sqrt{x^2 + y^2}}$. 

8. Let $\vec{F} = P \hat{i} + Q \hat{j}$ be the two dimensional vector field sketched below. Determine the signs of $P$, $Q$, $\frac{\partial Q}{\partial x}$ and $\frac{\partial Q}{\partial y}$ at the point $A$. 