I marked questions 1, 4, 5, 6, 9, and 10, at 3 points each plus up to 2 participation marks for 20 points total. The average was 16.9/20.

GENERALITIES

People generally did much better on this assignment. Well done!

- Surface parametrizations should have two independent parameters. Curves have one parameter. The number of parameters is the dimension of the space that you’re trying to describe. On the surface of the earth, you need two numbers, e.g. latitude and longitude, to tell people where you are. Along a highway, you need one (203 miles along Route 66). Stop and think next time about what exactly you are trying to parametrize and whether you have the correct number of parameters.
- When you write an equation of the tangent plane, don’t forget it’s an equation. You have to set some expression equal to something else (e.g. 0). If you just write $x + y - 2z$, that’s not an equation of a plane, but $x + y - 2z = 0$ is.
- Areas should be non-negative. Take care that your integrand is non-negative. Similarly don’t forget the absolute value signs in the expression $|\mathbf{r}_u \times \mathbf{r}_v|$ when you are computing area.
- A reminder that your midterm is next week. People did not perform well on the first midterm - take a bit of time to reflect on how you can improve your preparation strategy for this exam. Please ask on the forums or office hours if you have any lingering questions about any of the material.

QUESTION 1

This question caused the most trouble to people.

- You are asked to parametrize a surface. This should have two independent variables. Almost a third of you wrote down the equation of the circle that is the intersection of the plane and the cylinder. The question asks about the part of the plane which lies inside the cylinder. This is an entire disc shaped region, not just its boundary circle. Again, a surface will require two parameters because it’s two dimensional. If you only have one parameter, you are describing a curve, not a surface.
- The other common mistake was that many of you did not recognize this was an offset circle, with center at $(2,0)$. You tried to parametrize it with the standard polar or cylindrical coordinates (e.g. $x = r \cos \theta$ or even worse, combining with the previous error $x = 2 \cos \theta$), and then either ignored or somehow tried to incorporate the $4x$ into some expression for the radius. Parameters are the independent variables - you can’t have one parameter depending on the others.
Most people recognized how to set up the problem, but there was a lot of difficulty with carrying out the computation because the algebra was tricky. However, I want to highlight some conceptual errors.

- The radius should not be negative, so if you choose \( r = 8^{-1/2} x \sqrt{1 - x^2} \), then you really should have \( r = |8^{-1/2} x \sqrt{1 - x^2}| \) if \( x < 0 \), so the integral comes out positive. Many of you took an integral \( \int_{-1}^{1} \) over one integrand (which was an odd function), so that the part of the integral over \( \int_{-1}^{0} \) was negative and canceled out the contribution from \( \int_{0}^{1} \) and you ended up with net zero. That can’t be right, the area of that surface is clearly not zero.

- Many of you were oblivious to this, and halfway through your calculation decided to double the integral over \( \int_{0}^{1} \), so you never noticed the error. But had you carried out your original integral like some of your classmates, you would have ended up with zero. So be careful.

- Also, the square root means the positive square root here. So if you square an expression then take its square root, make sure you take the positive root, not the negative. People had expressions like \( \sqrt{(2x^2 - 3)^2} = 2x^2 - 3 \), but that quantity is negative for \( x \in [-1, 1] \). Instead, you should have \( \sqrt{(2x^2 - 3)^2} = 3 - 2x^2 \) for your domain. Many of you then ended up negative answers. Some people recognized that the answer should be positive since it’s area, and changed the sign at the end without understanding why. But this was the reason that you got the negative sign.

- A few people were confused with the additional symmetry about the \( x \)-axis - and tried to multiply the \( \int_{0}^{1} \) integral by 4. However, the bottom part of the curve doesn’t contribute anything additional because you are rotating the top part of the curve by the full \( 2\pi \).

Good luck on the midterm! Study hard!