Grader’s comments
nina@math.ubc.ca

Scoring

I marked questions 1 (4 points), 2 (b and c only) (4 points), 3 (2 points), 4 (2 points), 5 (2 points), and 8 (3 points), plus up to 3 participation marks for 20 points total. The average was 14.4/20.

Generalities

• As always, you must explain your answers to get credit.
• If you are asked to parametrize something, you need to state both the parametrization and the domain of the parameters.
• Many people did well on Question 1 and wrote good explanations of their answers, although there are still a few people who don’t understand these concepts. Curiously, a number of people answered question 1 completely correctly, then ignored what they had just written to revert to their old bad habits in later questions.
• Question 8 caused a lot of difficulties.
• You have a second midterm coming up in less than two weeks. Please ask questions in Piazza or office hours if you don’t understand something in the solution sets. You can also get help in the MLC. Some of your classmates are regular clients there.

Question 1

• If it’s a True/False question, please state which is your answer. A number of people did not do this, and wrote long paragraphs where it wasn’t clear what they were trying to prove.
• In a T/F question, the easiest way to show something is false is to state a counterexample. Also, in homework, if you want to prove something is true, you should include references from the book or lecture notes. (E.g., By Theorem 2, we know that ...)
• Some people said the domain $D$ was simply connected. This is not one of the given assumptions. Remember that simply connected is NOT the same as being connected, or open, or anything like that. If you don’t know the definition of simply connected, review the Notes.
• ‘Not simply connected’ does not imply ‘not conservative’. It just means you can’t use the screening test. The field may or may not be conservative on a non-simply-connected domain.
• Some people quoted Theorem 2.4.7 from clp4 as justification for concluding that $\vec{F}$ is conservative. Notice that that theorem only holds for domain equal to all of $\mathbb{R}^2$ or $\mathbb{R}^3$. The HW question concerns a open set of $\mathbb{R}^2$ or $\mathbb{R}^3$, not necessarily the whole space.
• Being simply connected is not a property of $\vec{F}$ or a curve $C$. It is a property of the region on which $\vec{F}$ is defined, or a region which contains $C$. It makes no sense whatsoever to say things like ‘$\vec{F}$ is not simply connected’.

1
Question 2

- Stating the domain of the parameters is part of giving a parametrization. Don’t forget.
- Some people managed to compute the integral in Part c, although it was long. That’s fine. But it’s easier to prove it with the theorems.
- To show Part c using theoretical arguments, you need to clearly state a new simply connected domain that you will be working on, as in the solutions. The original domain $\mathbb{R}^2 \setminus \{(0,0)\}$ is NOT simply connected so you cannot use that.
- It does not make sense to say things like ’$C_2$ is simply connected’. Also, the new domain must include $C_2$, so in particular, you cannot use the disk inside $C_2$ as your new domain.

Question 3

This was generally well done. It was quite long, but the techniques required were not new. Some people had errors and ended up with stray expressions like $\arctan(y/x)$ in their proposed potential. You should know that this can’t be correct - remember that $\arctan(y/x)$ (essentially, the polar coordinate $\theta$) is not defined on all of $\mathbb{R}^2 \setminus \{(0,0)\}$, so it’s not a valid function there. In order for $\phi$ to be a valid potential, it has to be defined on your domain.

Question 4

This was also straightforward and generally well done.
- If it asks you to parametrize a surface, you need to write the surface in terms of two parameters. Some people just left it as the equation of the plane, in terms of $x,y,z$. That’s not a parametrization.
- Do not forget to state the domain of the parameters.
- There were many other possible parametrizations.
- There were a few people who treated this as if it were a curve. A two dimensional surface needs two parameters. It cannot be written in terms of one parameter.

Question 5

This was also straightforward and generally well done.
- The vast majority of people used a parametrization $(u,v,\sqrt{u^2+v^2}+1)$, $u,v \in \mathbb{R}$, which is fine.
- If you used a parametrization in terms of $z$ or $r$, be careful that the domain of this parameter doesn’t include the negatives, because then you would be double covering your area.

Question 8

- For people who did this with rotated spherical coordinates as in Solution 2, the most common mistake was to let $\phi$ vary in $[\pi/3, 5\pi/3]$. I think you tried to solve for $2 \cos \phi \leq 1$. However, you only need the parameter to vary in half that angle, $[\pi/3, \pi]$ - otherwise you double cover the surface.
• Most people who tried to do this like Solution 1 of Question 7 did not consider that both the positive and negative square roots needed to be used, and that these would have different domains for their parameters.
• The cylindrical coordinate Solution 3 method was generally done correctly.
• Incorrect solutions included a variation on method 1, but with \( x \) and \( y \) as independent parameters and \( z \) as the dependent function. Again you would have to divide the surface and deal with the domains separately.
• By far the most common error was to try to use standard spherical coordinates:
  \[(2 \sin \phi \cos \theta, 2 \sin \phi \sin \theta, 2 \cos \phi),\]
  with some interval domains like \( \pi/3 \leq \phi \leq 5\pi/3 \), you got from trying to solve \( x \leq 1 \). However, this does not give the correct surface. I think it gives you some shape like an orange with a wedge removed, instead of an orange with the side sliced off straight.