Math 317 Calculus IV  Formulas and rules for final exam

1. For a curve \( \mathbf{r}(t) \), \( s = \int_0^t |\mathbf{r}'(\tau)| d\tau \), \( \frac{ds}{dt} = |\mathbf{r}'| \), \( ds = |\mathbf{r}'(t)| dt \)

2. \( \mathbf{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|} \), \( \mathbf{N} = \frac{\mathbf{T}'}{|\mathbf{T}'|} \), \( \mathbf{B} = \mathbf{T} \times \mathbf{N} \), \( \frac{d\mathbf{T}}{ds} = \kappa \mathbf{N} \), \( \frac{d\mathbf{N}}{ds} = -\kappa \mathbf{T} + \tau \mathbf{B} \), \( \frac{d\mathbf{B}}{ds} = -\tau \mathbf{N} \)

3. \( \kappa = \frac{|d\mathbf{T}|}{|\mathbf{r}'|} = |\mathbf{r}' \times \mathbf{r}''|/|\mathbf{r}'|^3 \), \( \mathbf{r}' \times \mathbf{r}'' = |\mathbf{r}'|^3 \kappa \mathbf{B} \), \( \mathbf{B} = \mathbf{r}' \times \mathbf{r}''/|\mathbf{r}' \times \mathbf{r}''| \), \( \tau = (\mathbf{r}' \times \mathbf{r}'' \cdot \mathbf{r}''')/|\mathbf{r}' \times \mathbf{r}''|^2 \)

4. For \( \mathbf{r} = (x(t), y(t)) \), \( \kappa(t) = \frac{|x'y'' - x''y'|}{(x')^2 + (y')^2} \); For \( y = f(x) \), \( \kappa(x) = \frac{|f''(x)|}{1 + (f'(x))^2} \)

5. For a surface \( S \) given by \( \mathbf{r}(u, v) : D \to \mathbb{R}^3 \), \( \text{Area}(S) = \iint_S dS = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| du dv \)
   If it has mass density \( \rho \), then its mass is \( \iint_S \rho dS = \iint_D \rho(\mathbf{r}) |\mathbf{r}_u \times \mathbf{r}_v| du dv \)
   \( \iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n} dS = \pm \iint_D \mathbf{F}(\mathbf{r}) \cdot (\mathbf{r}_u \times \mathbf{r}_v) du dv \)

6. For a graph \( S \) given by \( z = f(x, y) \), \( (x, y) \in D \), \( \text{Area}(S) = \iint_S dS = \iint_D \sqrt{1 + f_x^2 + f_y^2} \, dx \, dy \)
   Its mass is \( \iint_S \rho dS = \iint_D \rho(\mathbf{r}) \sqrt{1 + f_x^2 + f_y^2} \, dx \, dy \)
   \( \iint_S \mathbf{F} \cdot d\mathbf{S} = \pm \iint_D \mathbf{F}(\mathbf{r}) \cdot (-f_x, -f_y, 1) \, dx \, dy \)

7. Surface of revolution \( S : r = f(z) \), \( a \leq z \leq b \), \( \text{Area}(S) = \iint_S dS = \int_a^b 2\pi f(z) \sqrt{1 + |f'(z)|^2} \, dz \)

8. Divergence theorem: For a solid region \( V \) with boundary surface \( S \), \( \iiint_V \text{div} \mathbf{F} \, dV = \iint_S \mathbf{F} \cdot d\mathbf{S} \)

9. Green's theorem: \( \int_C F_1 \, dx + F_2 \, dy = \iint_D (\partial_2 F_2 - \partial_1 F_1) \, dA \)

10. Stokes' theorem: For a surface \( S \) with boundary curve \( C \), \( \oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S} \)

Rules for the Exam:

1. Bring a photo ID for the inspection of the invigilator.

2. During the exam, people may be relocated with no reasons.

3. From five minutes before the end of the exam, you cannot hand in your exam any more and should wait in your seat until the end of the exam.

4. When the invigilator says that the exam is over, you should **stop writing and remain seated**. Please pass your exam to the nearest aisle.

5. Do not discuss before you leave the room, since your neighbor may change her/his solutions after hearing your conversation.

6. You are not allowed to leave until the invigilator has collected all exams and says that you can leave.