Be sure that this examination has 10 pages including this cover

The University of British Columbia
Sessional Examinations - April 2003

Mathematics 317
Calculus IV

Closed book examination

Name ______________________________ Signature ______________________________

Student Number _______________ Instructor’s Name _______________

Section Number ___________________

Time: 2\(\frac{1}{2}\) hours

Special Instructions:
Calculators are permitted.
One 8\(\frac{1}{2}\)” × 11” two sided cheat sheet is permitted.

Rules Governing Formal Examinations

1. Each candidate must be prepared to produce, upon request, a library/AMS card for identification.

2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

3. No candidate shall be permitted to enter the examination room after the expiration of one half hour from the scheduled starting time, or to leave during the first half hour of the examination.

4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

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1. Find the field line of the vector field $\vec{F} = 2y \hat{i} + \frac{x}{y} \hat{j} + e^y \hat{k}$ that passes through $(1, 1, e)$. 
2. Let \( \vec{F} = e^x \sin y \hat{i} + [ae^x \cos y + bx] \hat{j} + cx \hat{k} \). For which values of the constants \( a, b, c \) is \( \int_C \vec{F} \cdot d\vec{r} = 0 \) for all closed paths \( C \)?
Let \( \vec{F} = \frac{x}{x^2+y^2} \mathbf{i} + \frac{y}{x^2+y^2} \mathbf{j} + x^3 \mathbf{k} \). Let \( P \) be the path which starts at \((1,0,0)\), ends at \((\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2} \ln 2)\) and follows
\[
x^2 + y^2 = 1 \quad xe^z = 1
\]
Find the work done in moving a particle along \( P \) in the field \( \vec{F} \).
4. Let the thin shell $S$ consist of the part of the surface $z^2 = 2xy$ with $x \geq 1$, $y \geq 1$ and $z \leq 2$. Find the mass of $S$ if it has surface density given by $\rho(x, y, z) = 3z$ kg per unit area.

\[\int_{\Gamma} \rho(x, y, z) \, dS\]
5. Let $S$ be the portion of the paraboloid $x = y^2 + z^2$ that satisfies $x \leq 2y$. Its unit normal vector $\mathbf{n}$ is so chosen that $\mathbf{n} \cdot \mathbf{i} > 0$. Find the flux of $\mathbf{F} = 2\mathbf{i} + z\mathbf{j} + y\mathbf{k}$ out of $S$. 

Continued on page 7
6. Let $S$ be the portion of the hyperboloid $x^2 + y^2 - z^2 = 1$ between $z = -1$ and $z = 1$. Find the flux of $\vec{F} = (x + e^{yz})\hat{i} + (2yz + \sin(xz))\hat{j} + (xy - z - z^2)\hat{k}$ out of $S$ (away from the origin).
7. Evaluate \( \iiint_S \nabla \times \vec{F} \cdot \hat{n} \, dS \) where \( \vec{F} = yi + 2zj + 3xk \) and \( S \) is the surface \( z = \sqrt{1 - x^2 - y^2} \), \( z \geq 0 \) and \( \hat{n} \) is a unit normal to \( S \) obeying \( \hat{n} \cdot \hat{k} \geq 0 \).
8. The following statements may be true or false. Decide which. If true, give a proof. If false, provide a counter-example.

a) If \( f \) is any smooth function defined in \( \mathbb{R}^3 \) and if \( C \) is any circle, then \( \int_C \nabla f \cdot d\mathbf{r} = 0 \).

b) There is a vector field \( \mathbf{F} \) that obeys \( \nabla \times \mathbf{F} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k} \).