No calculators, books, notebooks or any other written materials are allowed

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Rules for the exam:

1. You should sit in odd numbered seats from the nearest aisle. (A seat next to an aisle is number 1.) No two persons are allowed to sit next to each other.

2. From five minutes before the end of the exam, you cannot hand in your exam any more and should wait in your seat until the end of the exam.

3. When the invigilator says that the exam is over, you should stop writing and immediately pass your exam to the end of your row. The invigilator will collect the exams in the aisles.

4. You are not allowed to leave until the invigilator has collected all exams and says that you can leave.

5. Any conversation between students during the exam and the exam collection time is considered cheating. Writing after the end of exam is considered cheating.
1. The vector field $\vec{F}(x, y, z) = Ax^3 y^2 z \hat{i} + (z^3 + Bx^4 yz) \hat{j} + (3yz^2 - x^4 y^2) \hat{k}$ is conservative on $\mathbb{R}^3$.

(a) Find the values of the constants $A$ and $B$.

(b) Find a potential function $\phi$ such that $\vec{F} = \nabla \phi$ on $\mathbb{R}^3$ and $\phi(0, 0, 0) = 2$.

(c) If $C$ is the curve $y = -x$, $z = x^2$ from $(0, 0, 0)$ to $(1, -1, 1)$, evaluate $I = \int_C \vec{F} \cdot d\vec{r}$.
(10 pt) 2. Let $\vec{F}(x, y) = (x^4 - 8x^3y + 2xy^3 + y^4)\hat{i} + (3x - 2x^4 + 3x^2y^2 + 4xy^3 + y^4)\hat{j}$.

(a) Compute $\int_{C_1} \vec{F} \cdot d\vec{r}$ where $C_1$ is the straight line segment from $(-1, -1)$ to $(1, 1)$.

(b) Compute $\int_{C_2} \vec{F} \cdot d\vec{r}$ where $C_2$ is the arc from $(-1, -1)$ to $(1, 1)$ along the lower-right half circle $x^2 + y^2 = 2$, $y \leq x$. 
3. (a) Let $\vec{F}(x, y, z) = (xe^y, \sin(z)\cos(x), x^2 + y^2 + z^2)$. Compute $\text{curl}\, \vec{F}$ and $\text{div}\, \vec{F}$.

(b) Find the mass of a wire in the shape of the helix $x = t, y = \cos t, z = \sin t, 0 \leq t \leq 2\pi$, if its density is $\rho(x, y, z) = x^2 + y^2 + z^2$. 

4. (a) A surface $S$ has the parametric equation $\mathbf{r} = (u^2, u - v^2, v^2)$ with $0 \leq u \leq \sqrt{3}$ and $1 \leq v \leq 3$. Find its tangent plane at the point $P(1, -3, 4)$.

(b) Set up the integral, but do not evaluate it, for the area of the part of the paraboloid $x^2 + z^2 + y = 3$ that satisfies $y \geq 2x$. 
Formulas for MT2

1. For a curve \( r(t) \), arclength \( s = \int_0^t |r'(\tau)|d\tau \), \( \frac{ds}{dt} = |r'| \), \( ds = |r'(t)|dt \)

2. \( T = \frac{r'}{|r'|}, \quad N = \frac{T'}{|T'|}, \quad B = T \times N \)

3. \( \kappa = \left| \frac{dT}{ds} \right| = \frac{|T'|}{|r'|} = \frac{|r' \times r''|}{|r'|^3}, \quad \kappa N = \frac{dT}{ds} \)

4. For \( y = f(x) \), \( \kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}} \)

5. Green’s theorem: \( \int_C Pdx + Qdy = \iint_D (Q_x - P_y) dA \)

6. For a surface \( S \) given by \( r(u, v) : D \to \mathbb{R}^3 \), the surface area is \( \int_S dS = \iint_D |r_u \times r_v| dudv \)

7. For a graph \( S \) given by \( z = f(x, y), (x, y) \in D \), the surface area is \( \int_S dS = \iint_D \sqrt{1 + f_x^2 + f_y^2} dxdy \)

8. For a surface of revolution \( S \) given by \( r = f(z), a \leq z \leq b \), the surface area is \( \int_S dS = \int_a^b 2\pi f(z) \sqrt{1 + [f'(z)]^2} dz \)