4. (a) 

Thus

\[ \int_{C} x^{2}y^{2} \, dx + xy \, dy = \int_{C_{1} \cup C_{2} \cup C_{3}} x^{2}y^{2} \, dx + xy \, dy \]

\[ = \int_{C_{1}} x^{2}y^{2} \, dx + xy \, dy + \int_{C_{2}} x^{2}y^{2} \, dx + xy \, dy + \int_{C_{3}} x^{2}y^{2} \, dx + xy \, dy \]

\[ = \int_{C_{1}} x^{2}y^{2} \, dx + xy \, dy + \int_{C_{2}} x^{2}y^{2} \, dx + xy \, dy + \int_{C_{3}} x^{2}y^{2} \, dx + xy \, dy \]

\[ = \int_{C_{1}} x^{2}y^{2} \, dx + xy \, dy + \int_{C_{2}} x^{2}y^{2} \, dx + xy \, dy + \int_{C_{3}} x^{2}y^{2} \, dx + xy \, dy \]

\[ = \int_{C_{1}} x^{2}y^{2} \, dx + xy \, dy + \int_{C_{2}} x^{2}y^{2} \, dx + xy \, dy + \int_{C_{3}} x^{2}y^{2} \, dx + xy \, dy \]

(b) \( x^{2}y^{2} \, dx + xy \, dy = \int_{[0, 1]} \left( \frac{d}{d \gamma} (x) \cdot \frac{d}{d \gamma} (y) \right) \, dA = \int_{[0, 1]} (2x + 1) \, dx + \int_{[0, 1]} (2y + 1) \, dy \]

\[ = \frac{3}{2} + \frac{2}{3} = \frac{11}{6} \]

6. The region \( D \) enclosed by \( C \) is \([0, 1] \times [0, 2] \), so

\[ \int_{C} \cos y \, dx + x^{2} \sin y \, dy = \int_{[0, 1]} \left( \frac{d}{d \gamma} (x) \cdot \cos y \, d \gamma \right) \, dA = \int_{[0, 1]} \left( 2x + 1 \right) \, dx + \int_{[0, 1]} \left( 2y + 1 \right) \, dy \]

\[ = \frac{3}{2} + \frac{2}{3} = \frac{11}{6} \]

8. \( x^{4}y^{2} + 2x^{3}y^{3} \, dy = \int_{[0, 1]} \left( \frac{d}{d \gamma} (x) \cdot (2x^{3}y^{3}) \right) \, dA = \int_{[0, 1]} \left( 2x^{4}y^{3} + 6x^{3}y^{4} \right) \, dA \]

\[ = 2 \int_{[0, 1]} (x^{4}y^{3}) \, dA = 0 \]

10. \( x(1 - y) \, dx + (x^{2} + y^{2}) \, dy = \int_{[0, 1]} \left( \frac{d}{d \gamma} (x) \cdot (1 - y) \right) \, dA = \int_{[0, 1]} \left( 2x^{2} + 3y^{2} \right) \, dA \]

\[ = \int_{[0, 1]} \left( 2x^{2} + 3y^{2} \right) \, dA \]

\[ = \frac{3}{2} \int_{[0, 1]} \left( x^{2} \right) \, dx + \frac{3}{2} \int_{[0, 1]} \left( 1 \right) \, dx \]

\[ = \frac{3}{2} \left( \frac{1}{3} \right) + \frac{3}{2} \left( 1 \right) = \frac{2}{3} \]

12. \( F(x, y) = (x^{2} - y^{2}, e^{-x} - e^{y}) \) and the region \( D \) enclosed by \( C \) is given by \( \{(x, y) : -\pi/2 \leq x \leq \pi/2, 0 \leq y \leq \cos x\} \).

\( C \) is traversed clockwise, so \( -C \) gives the positive orientation.

\[ \int_{C} F \cdot dr = -\int_{C} (x^{2} - y^{2}) \, dx + (e^{-x} - e^{y}) \, dy = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

\[ = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

\[ = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

\[ = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

\[ = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

\[ = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

\[ = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

\[ = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

\[ = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

\[ = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

\[ = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

\[ = -\int_{[0, 1]} \left( x^{2} + e^{y} - \frac{1}{2} e^{-x} - e^{y} \right) \, dA \]

14. \( F(x, y) = (\sqrt{x^{2} + 1}, \tan^{-1} x) \) and the region \( D \) enclosed by \( C \) is given by \( \{(x, y) : 0 \leq y \leq 1, 1 \leq y \leq y \leq y \leq 1\} \).

\( C \) is oriented positively, so

\[ \int_{C} F \cdot dr = \int_{[0, 1]} \left( \frac{d}{d \gamma} (x) \cdot (\sqrt{x^{2} + 1}) + \frac{d}{d \gamma} (\tan^{-1} x) \right) \, dA \]

\[ = \int_{[0, 1]} \left( \frac{1}{1 + x^{2}} \right) \, dx + \int_{[0, 1]} \left( \frac{1}{1 + x^{2}} \right) \, dx \]

\[ = \int_{[0, 1]} \left( \frac{1}{1 + x^{2}} \right) \, dx \]

\[ = \left[ \tan^{-1} x - \frac{1}{2} \ln(1 + x^{2}) \right]_{0}^{1} = \frac{\pi}{4} - \frac{1}{2} \ln 2 \]
18. By Green's Theorem, \[ W = \oint_D \mathbf{F} \cdot d\mathbf{r} = \int_C x \, dx + (x^2 + 3y^2) \, dy = \int_C (3x^2 + 3y^2) \, dA, \] where \( D \) is the semicircular region bounded by \( C \). Converting to polar coordinates, we have \[ W = 3 \int_0^\pi \left( \frac{1}{4} r^2 \right) \, r \, dr \cdot d\theta = 3\pi \left( \frac{1}{8} \right) = \frac{3\pi}{8}. \]

22. By Green's Theorem, \[ \frac{1}{2} \int_D x^2 \, dA = \frac{1}{2} \oint_C x \, dx + y \, dy \quad \text{and} \quad -\frac{1}{2} \int_D y^2 \, dA = -\frac{1}{2} \oint_C y \, dx + x \, dy. \]

24. Here \( A = \frac{1}{4}a^2 \) and \( C = C_1 + C_2 + C_3 \), where \( C_1: x = a \), \( C_2: y = 0 \), \( 0 \leq y \leq b \), \( C_3: x = a \), \( y = 0 \) to \( x = a \), \( y = b \) to \( x = 0 \). Then \[ \oint_D x^2 \, dy = \int_{C_1} x^2 \, dy + \int_{C_2} x^2 \, dy + \int_{C_3} x^2 \, dy = 0 + \int_0^b a^2 \, dy + \int_a^b (x^2) \left( \frac{dy}{dx} \right) \, dx = a^2b + \frac{1}{2} \left( \frac{1}{2} x^3 \right)_a^b = a^2b - \frac{1}{8} a^3b = \frac{1}{2} a^2b. \]

Similarly, \[ \oint_D y^2 \, dx = \int_{C_1} y^2 \, dx + \int_{C_2} y^2 \, dx + \int_{C_3} y^2 \, dx = 0 + \int_0^b y^2 \, dx + \int_a^b (y^2) \left( \frac{dx}{dy} \right) \, dy = \frac{2}{3} y^3 \bigg|_0^b = \frac{2}{3} b^3. \]

Thus \( y = \frac{1}{2}a \) and \( y = \frac{1}{2}a \) and \( x = \frac{1}{2}a \) and \( y = -\frac{1}{2}a \), so \( (\theta, y) = \left( \frac{\pi}{2}, \frac{3}{2}b \right) \).

26. \( P \) and \( Q \) have continuous partial derivatives on \( \mathbb{R}^2 \), so by Green's Theorem we have \[ \int_D \oint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA = \int_D \oint_D (3-1) \, dA = 2 \int_D 2 \cdot dA = 2 \cdot A(D) = 2 \cdot \pi = 12. \]