Exercises

Determine whether the points \( P \) and \( Q \) lie on the given surface.

\( \mathbf{r}(u, v) = (2u + 3v, 1 + 5u - v, 2 + u + v) \)
\( P(7, 10, 4), \ Q(5, 22, 5) \)

\( \mathbf{r}(u, v) = (u + v, u^2 - v, u + v^2) \)
\( P(3, -1, 5), \ Q(-1, 3, 4) \)

3–6 Identify the surface with the given vector equation.

3. \( \mathbf{r}(u, v) = (u + v) \mathbf{i} + (3 - v) \mathbf{j} + (1 + 4u + 5v) \mathbf{k} \)
4. \( \mathbf{r}(u, v) = 2 \sin u \mathbf{i} + 3 \cos u \mathbf{j} + v \mathbf{k}, \quad 0 \leq v \leq 2 \)
5. \( \mathbf{r}(s, t) = (s, t, t^2 - s^3) \)
6. \( \mathbf{r}(x, y) = (x \sin 2t, s^2, s \cos 2t) \)

7–12 Use a computer to graph the parametric surface. Get a printout and indicate on it which grid curves have \( u \) constant and which have \( v \) constant.

7. \( \mathbf{r}(u, v) = \langle u^2, v^3, u + v \rangle, \quad -1 \leq u \leq 1, -1 \leq v \leq 1 \)
8. \( \mathbf{r}(u, v) = \langle u, v^3, -v \rangle, \quad -2 \leq u \leq 2, -2 \leq v \leq 2 \)
9. \( \mathbf{r}(u, v) = \langle u \cos v, u \sin v, u^2 \rangle, \quad -1 \leq u \leq 1, 0 \leq v \leq 2\pi \)
10. \( \mathbf{r}(u, v) = \langle u, \sin(u + v), \sin v \rangle, \quad -\pi \leq u \leq \pi, -\pi \leq v \leq \pi \)
11. \( x = \sin v, \quad y = \cos u \sin 4v, \quad z = \sin 2u \sin 4v, \quad 0 \leq u \leq 2\pi, -\pi/2 \leq v \leq \pi/2 \)
12. \( x = \sin u, \quad y = \cos u \sin v, \quad z = \sin v, \quad 0 \leq u \leq 2\pi, 0 \leq v \leq 2\pi \)

13–18 Match the equations with the graphs labeled I–VI and give reasons for your answers. Determine which families of grid curves have \( u \) constant and which have \( v \) constant.

13. \( \mathbf{r}(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + v \mathbf{k} \)
14. \( \mathbf{r}(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + v \mathbf{k}, \quad -\pi \leq u \leq \pi \)
15. \( \mathbf{r}(u, v) = \sin v \mathbf{i} + \cos u \sin 2v \mathbf{j} + \sin u \sin 2v \mathbf{k} \)
16. \( x = (1 - u)(3 + \cos v) \cos 4\pi u, \quad y = (1 - u)(3 + \cos v) \sin 4\pi u, \quad z = 3u + (1 - u) \sin v \)
17. \( x = \cos^3 u \cos^3 v, \quad y = \sin^3 u \cos^3 v, \quad z = \sin^3 v \)
18. \( x = (1 - |u|) \cos v, \quad y = (1 - |u|) \sin v, \quad z = u \)

Graphing calculator or computer required

19–26 Find a parametric representation for the surface.

19. The plane through the origin that contains the vectors \( \mathbf{i} - \mathbf{j} \) and \( \mathbf{j} - \mathbf{k} \)
20. The plane that passes through the point \((0, -1, 5)\) and contains the vectors \(\langle 2, 1, 4 \rangle\) and \(\langle -3, 2, 5 \rangle\)
21. The part of the hyperboloid \(4z^2 - 4y^2 - z^2 = 4\) that lies in front of the \(yz\)-plane
22. The part of the ellipsoid \(x^2 + 2y^2 + 3z^2 = 1\) that lies to the left of the \(xz\)-plane
23. The part of the sphere \(x^2 + y^2 + z^2 = 4\) that lies above the cone \(z = \sqrt{x^2 + y^2}\)
24. The part of the sphere \(x^2 + y^2 + z^2 = 16\) that lies between the planes \(z = -2\) and \(z = 2\)
25. The part of the cylinder \(y^2 + z^2 = 16\) that lies between the planes \(x = 0\) and \(x = 5\)

1. Homework Hints available at stewartcalculus.com
26. The part of the plane $z = x + 3$ that lies inside the cylinder $x^2 + y^2 = 1$

27-28 Use a computer algebra system to produce a graph that looks like the given one.

29. Find parametric equations for the surface obtained by rotating the curve $y = e^{-x}$, $0 \leq x \leq 3$, about the $x$-axis and use them to graph the surface.

30. Find parametric equations for the surface obtained by rotating the curve $x = 4y^2 - y^4$, $-2 \leq y \leq 2$, about the $y$-axis and use them to graph the surface.

31. (a) What happens to the spiral tube in Example 2 (see Figure 5) if we replace $\cos u$ by $\sin u$ and $\sin u$ by $\cos u$? (b) What happens if we replace $\cos u$ by $2\cos u$ and $\sin u$ by $\sin 2u$?

32. The surface with parametric equations

$$
\begin{align*}
x &= 2\cos \theta + r \cos(\theta/2) \\
y &= 2\sin \theta + r \cos(\theta/2) \\
z &= r \sin(\theta/2)
\end{align*}
$$

where $-\frac{1}{2} \leq r \leq \frac{1}{2}$ and $0 \leq \theta \leq 2\pi$, is called a Möbius strip. Graph this surface with several viewpoints. What is unusual about it?

33-36 Find an equation of the tangent plane to the given parametric surface at the specified point.

33. $x = u + v, \quad y = 3u^2, \quad z = u - v; \quad (2, 3, 0)$

34. $x = u^2 + 1, \quad y = v^3 + 1, \quad z = u + v; \quad (5, 2, 3)$

35. $r(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + v \mathbf{k}; \quad u = 1, \quad v = \pi/3$

36. $r(u, v) = \sin u \mathbf{i} + \cos u \mathbf{j} + \sin v \mathbf{k}; \quad u = \pi/6, \quad v = \pi/6$

37-38 Find an equation of the tangent plane to the given parametric surface at the specified point. Graph the surface and the tangent plane.

37. $r(u, v) = u^2 \mathbf{i} + 2u \sin v \mathbf{j} + u \cos v \mathbf{k}; \quad u = 1, \quad v = 0$

38. $r(u, v) = (1 - u^2 - v^2) \mathbf{i} - v \mathbf{j} - u \mathbf{k}; \quad (-1, -1, -1)$

39-50 Find the area of the surface.

39. The part of the plane $3x + 2y + z = 6$ that lies in the first octant

40. The part of the plane with vector equation $r(u, v) = (u + v, 2 - 3u, 1 + u - v)$ that is given by $0 \leq u \leq 2, -1 \leq v \leq 1$

41. The part of the plane $x + 2y + 3z = 1$ that lies inside the cylinder $x^2 + y^2 = 3$

42. The part of the cone $z = \sqrt{x^2 + y^2}$ that lies between the plane $y = x$ and the cylinder $y^2 = x^2$

43. The surface $z = \frac{1}{2}(x^{3/2} + y^{1/2}), 0 \leq x \leq 1, 0 \leq y \leq 1$

44. The part of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with vertices $(0, 0), (0, 1), \text{and} (2, 1)$

45. The part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$

46. The part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$

47. The part of the surface $y = 4x + z^2$ that lies between the planes $x = 0, x = 1, z = 0,$ and $z = 1$

48. The helicoid (or spiral ramp) with vector equation $r(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + v \mathbf{k}, 0 \leq u \leq 1, 0 \leq v \leq \pi$

49. The surface with parametric equations $x = u^2, y = uv, z = \frac{1}{2}v^2, 0 \leq u \leq 1, 0 \leq v \leq 2$

50. The part of the sphere $x^2 + y^2 + z^2 = b^2$ that lies inside the cylinder $x^2 + y^2 = a^2$, where $0 < a < b$

51. If the equation of a surface $S$ is $z = f(x, y)$, where $x^2 + y^2 \leq R^2$, and you know that $|f_x| \leq 1$ and $|f_y| \leq 1$, what can you say about $A(S)$?

52-53 Find the area of the surface correct to four decimal places by expressing the area in terms of a single integral and using your calculator to estimate the integral.

52. The part of the surface $z = \cos(x^2 + y^2)$ that lies inside the cylinder $x^2 + y^2 = 1$

53. The part of the surface $z = e^{-x^2-y^2}$ that lies above the disk $x^2 + y^2 \leq 4$

54. Find, to four decimal places, the area of the part of the surface $z = (1 + x^2)/(1 + y^2)$ that lies above the square $|x| + |y| \leq 1$. Illustrate by graphing this part of the surface.

55. (a) Use the Midpoint Rule for double integrals (see Section 15.1) with six squares to estimate the area of the surface $z = 1/(1 + x^2 + y^2)$, $0 \leq x \leq 6, 0 \leq y \leq 4$. 