5-10 Use Green’s Theorem to evaluate the line integral along the given positively oriented curve.

\[ \int_C (y \cos x - xy \sin x, xy + x \cos x) \cdot \mathbf{dr}, \]
\[ C \text{ is the triangle from } (0, 0) \text{ to } (2, 0) \text{ to } (0, 0) \]

5. \[ \int_C xy^2 \, dx + 2x^2y \, dy, \]
\[ C \text{ is the triangle with vertices } (0, 0), (2, 2), \text{ and } (2, 4) \]

6. \[ \int_C \cos x \, dy + x^2 \sin x \, dx, \]
\[ C \text{ is the rectangle with vertices } (0, 0), (5, 0), (5, 2), \text{ and } (0, 2) \]

7. \[ \int_C (y + e^{-x}) \, dx + (2x + \cos y) \, dy, \]
\[ C \text{ is the boundary of the region enclosed by the parabolas } y = x^2 \text{ and } x = y^2 \]

8. \[ \int_C y^2 \, dx + 2x^2 \, dy, \]
\[ C \text{ is the ellipse } x^2 + 2y^2 = 2 \]

9. \[ \int_C y^2 \, dx - x^3 \, dy, \]
\[ C \text{ is the circle } x^2 + y^2 = 4 \]

10. \[ \int_C (1 - y^2) \, dx + (x^3 + e^y) \, dy, \]
\[ C \text{ is the boundary of the region between the circles } x^2 + y^2 = 4 \text{ and } x^2 + y^2 = 9 \]

11-14 Use Green’s Theorem to evaluate \( \int_C F \cdot dr \). (Check the orientation of the curve before applying the theorem.)

11. \( F(x, y) = (\cos x - xy \sin x, xy + x \cos x) \),
\[ C \text{ is the triangle from } (0, 0) \text{ to } (2, 0) \text{ to } (0, 0) \]

12. \( F(x, y) = (e^x + y^2, e^{-y} + x^2) \),
\[ C \text{ consists of the arc of the curve } y = \cos x \text{ from } (\pi/2, 0) \text{ to } (\pi/2, 0) \text{ to } \text{ and the line segment from } (\pi/2, 0) \text{ to } (\pi/2, 0) \]

13. \( F(x, y) = (y - \cos y, x \sin y) \),
\[ C \text{ is the circle } (x - 3)^2 + (y - 4)^2 = 4 \text{ oriented clockwise} \]

14. \( F(x, y) = (\sqrt{x^2 + 1}, \tan^{-1} x) \),
\[ C \text{ is the triangle from } (0, 0) \text{ to } (1, 0) \text{ to } (0, 0) \]

15-16 Verify Green’s Theorem by using a computer algebra system to evaluate both the line integral and the double integral.

15. \( P(x, y) = y^2e^y, \quad Q(x, y) = x^2e^y \),
\[ C \text{ consists of the line segment from } (-1, 1) \text{ to } (1, 1) \text{ followed by the arc of the parabola } y = 2 - x^2 \text{ from } (1, 1) \text{ to } (-1, 1) \]

16. \( P(x, y) = 2x - x^2y^4, \quad Q(x, y) = x^3y^4 \),
\[ C \text{ is the ellipse } 4x^2 + y^2 = 4 \]

17. Use Green’s Theorem to find the work done by the force \( F(x, y) = x(x + y) \mathbf{i} + xy^2 \mathbf{j} \) in moving a particle from the origin along the x-axis to (1, 0), then along the line segment to (0, 1), and then back to the origin along the y-axis.

18. A particle starts at the point \((-2, 0)\), moves along the x-axis to \((2, 0)\), and then along the semicircle \( y = \sqrt{4 - x^2} \) to the starting point. Use Green’s Theorem to find the work done on this particle by the force field \( F(x, y) = (x, x^3 + 3xy^2) \).

19. Use one of the formulas in 3 to find the area under one arch of the cycloid \( x = t - \sin t, y = 1 - \cos t \) \( \rho \).

20. If a circle \( C \) with radius 1 rolls along the outside of the circle \( x^2 + y^2 = 16 \), fixed point \( P \) on \( C \) traces out a curve called an epicycloid, with parametric equations \( x = 5 \cos t - \cos 5t, y = 5 \sin t - \sin 5t \). (Graph the epicycloid and use 3 to find the area it encloses.)

21. (a) If \( C \) is the line segment connecting the point \((x_1, y_1)\) to the point \((x_2, y_2)\), show that
\[ \int_C x \, dy - y \, dx = x_1y_2 - y_1x_2 \]
(b) If the vertices of a polygon, in counterclockwise order, are \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\), show that the area of the polygon is
\[ A = \frac{1}{2} \left[ (x_1y_2 - y_1x_2) + (x_2y_3 - x_1y_3) + \cdots + (x_{n-1}y_n - x_ny_{n-1}) + (x_ny_1 - x_1y_n) \right] \]
(c) Find the area of the pentagon with vertices \((0, 0), (2, 1), (1, 3), (0, 2), \) and \((-1, 1)\).

22. Let \( D \) be a region bounded by a simple closed path \( C \) in the \( xy \)-plane. Use Green’s Theorem to prove that the coordinates of the centroid \((\bar{x}, \bar{y})\) of \( D \) are
\[ \bar{x} = \frac{1}{2A} \int_C x^2 \, dy \quad \bar{y} = -\frac{1}{2A} \int_C y^2 \, dx \]
where \( A \) is the area of \( D \).

23. Use Exercise 22 to find the centroid of a quarter-circular region of radius \( a \).

24. Use Exercise 22 to find the centroid of the triangle with vertices \((0, 0), (a, 0), \) and \((a, b)\), where \( a > 0 \) and \( b > 0 \).

25. A plane lamina with constant density \( \rho(x, y) = \rho \) occupies a region in the \( xy \)-plane bounded by a simple closed path \( C \). Show that its moments of inertia about the axes are
\[ I_x = \frac{\rho}{3} \int_C x^3 \, dy \quad I_y = \frac{\rho}{3} \int_C y^3 \, dx \]

26. Use Exercise 25 to find the moment of inertia of a circular disk of radius \( a \) with constant density \( \rho \) about a diameter. (Compare with Example 4 in Section 15.5.)

27. Use the method of Example 5 to calculate \( \int_C F \cdot dr \), where
\[ F(x, y) = \frac{2xy \mathbf{i} + (y^2 - x^2) \mathbf{j}}{(x^2 + y^2)^2} \]
and \( C \) is any positively oriented simple closed curve that encloses the origin.

28. Calculate \( \int_C F \cdot dr \), where \( F(x, y) = (x^2 + y, 3x - y^2) \) and \( C \) is the positively oriented boundary curve of a region \( D \) that has area 6.

29. If \( F \) is the vector field of Example 5, show that \( \int_F F \cdot dr = 0 \) for every simple closed path that does not pass through or enclose the origin.