1. (§2.1#9) Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$. What value(s) of $k$, if any, will make $AB = BA$?

2. (§2.1#10) Let $A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 3 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & -5 \\ 2 & 1 \end{bmatrix}$. Verify that $AB = AC$ and yet $B \neq C$.

3. (§2.1#15) The following questions concern arbitrary matrices $A$, $B$, and $C$ for which the indicated sums and products are defined. Mark each statement True or False. Justify each answer.

(a) If $A$ and $B$ are $2 \times 2$ matrices with columns $\vec{a}_1$, $\vec{a}_2$ and $\vec{b}_1$, $\vec{b}_2$, respectively, then $AB = [\vec{a}_1 \vec{b}_1 \quad \vec{a}_2 \vec{b}_2]$.

(b) Each column of $AB$ is a linear combination of the columns of $B$ using weights from the corresponding column of $A$.

(c) $AB + AC = A(B + C)$

(d) $A^T + B^T = (A + B)^T$

(e) The transpose of a product of matrices equals the product of their transposes in the same order.

4. (§2.1#21) Suppose the last column of $AB$ is entirely zeros but $B$ itself has no column of zeros. What can be said about the columns of $A$?

5. (§2.1#22) Show that if the columns of $B$ are linearly dependent, then so are the columns of $AB$.

6. (§2.2#2) Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 \\ 8 & 5 \end{bmatrix}$.

7. (§2.2#9) Mark each statement True or False. Justify each answer.

(a) In order for a matrix $B$ to be the inverse of $A$, the equations $AB = I$ and $BA = I$ must both be true.

(b) If $A$ and $B$ are $n \times n$ and invertible, then $A^{-1}B^{-1}$ is the inverse of $AB$.

(c) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ab - cd \neq 0$, then $A$ is invertible.

(d) If $A$ is an invertible $n \times n$ matrix, then the equation $A\vec{x} = \vec{b}$ is consistent for each $\vec{b}$ in $\mathbb{R}^n$.

(e) (skip)

8. (§2.2#31) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$, if it exists.
9. (§2.2#37) Let \( A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix} \). Construct a \( 2 \times 3 \) matrix \( C \) (by trial and error) using only 1, \(-1\), and 0 as entries, such that \( CA = I_2 \). Compute \( AC \) and note that \( AC \neq I_3 \).

10. (§2.3#11) In the following questions, all matrices are \( n \times n \). Mark each statement True or False. Justify each answer.

   (a) If the equation \( A\vec{x} = \vec{0} \) has only the trivial solution, then \( A \) is row equivalent to the \( n \times n \) identity matrix.

   (b) If the columns of \( A \) span \( \mathbb{R}^n \), then the columns are linearly independent.

   (c) If \( A \) is an \( n \times n \) matrix, then the equation \( A\vec{x} = \vec{b} \) has at least one solution for each \( \vec{b} \) in \( \mathbb{R}^n \).

   (d) If the equation \( A\vec{x} = \vec{0} \) has a nontrivial solution, then \( A \) has fewer than \( n \) pivot positions.

   (e) If \( A^T \) is not invertible, then \( A \) is not invertible.

11. (§2.3#17) Can a square matrix with two identical columns be invertible? Why or why not?

12. (§2.3#26) Explain why the columns of \( A^2 \) span \( \mathbb{R}^n \) whenever the columns of an \( n \times n \) matrix \( A \) are linearly independent.

13. (§2.5#7,9) Let \( A = [\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3] \) and

\[
\vec{v}_1 = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -3 \\ 8 \\ -7 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -4 \\ 6 \\ -7 \end{bmatrix}, \quad \vec{p} = \begin{bmatrix} 6 \\ -10 \\ 11 \end{bmatrix}.
\]

   (a) How many vectors are in \( \{\vec{v}_1, \vec{v}_2, \vec{v}_3\} \)?

   (b) How many vectors are in \( \text{Col} \ A \)?

   (c) Is \( \vec{p} \) in \( \text{Col} \ A \)? Why or why not?

   (d) Is \( \vec{p} \) in \( \text{Nul} \ A \)? Why or why not?

14. (§2.5#18) Determine if the set of vectors

\[
\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 5 \\ 1 \\ -4 \end{bmatrix},
\]

is a basis for \( \mathbb{R}^3 \). Justify the answer.

15. (§2.5#24) The matrix \( A = \begin{bmatrix} 3 & -6 & 9 & 0 \\ 2 & -4 & 7 & 2 \end{bmatrix} \) has an echelon form \( \begin{bmatrix} 1 & -2 & 5 & 4 \\ 0 & 0 & 3 & 6 \end{bmatrix} \). Find a basis for \( \text{Col} \ A \) and a basis for \( \text{Nul} \ A \).

16. (§2.5#36) What can be said about \( \text{Nul} \ C \) when \( C \) is a \( 6 \times 4 \) matrix with linearly independent columns?