Rules for the exam:

1. Bring a photo ID for the inspection of the invigilator.

2. During the exam, people may be relocated with no reasons.

3. From five minutes before the end of the exam, you cannot hand in your exam any more and should wait in your seat until the end of the exam.

4. When the invigilator says that the exam is over, you should **stop writing and remain seated**. Please pass your exam to the nearest aisle.

5. Do not discuss before you leave the room, since your neighbor may change her/his solutions after hearing your conversation.

6. You are not allowed to leave until the invigilator has collected all exams and says that you can leave.
(8pt) 1. Solve the following linear system and write the solution in parametric vector form.

\[
\begin{align*}
2x_1 + 4x_2 + 2x_3 + 2x_4 &= 6 \\
x_1 + 2x_2 + x_3 &= 3 \\
x_1 + x_2 - x_3 &= 3
\end{align*}
\]
(7pt) 2. Write down all equations, if any, satisfied by the components $b_1, b_2, b_3$ of \( \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \) that is in the Span of

\[
\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}.
\]

Is the Span the entire $\mathbb{R}^3$? Why or why not?
3. (a) Find all values of $h$, such that the vectors

$$
\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ h \\ 3 \\ 5 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}
$$

are linearly dependent.

(b) Choose a value of $h$ such that $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ are linearly dependent, and write down a linear dependence relation.
4. Suppose \( T : \mathbb{R}^3 \to \mathbb{R}^2 \) is a linear transformation such that

\[
T \left( \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad T \left( \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad T \left( \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.
\]

(a) What is \( T \left( \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \right) \)?

(b) What is \( T \left( \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) \)?  
*Hint.* Write \( \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \) as a linear combination.
(10pt) 5. Circle the correct answers. No justifications necessary.

(a) True or false? A $5 \times 7$ matrix has seven rows.

(b) True or false? The matrix
\[
\begin{bmatrix}
1 & 0 & -1 & 0 \\
0 & 1 & -2 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]
is in reduced echelon form.

(c) True or false? In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations.

(d) True or false? A linear system is unique if and only if it has exactly one solution.

(e) True or false? The equation $A\vec{x} = \vec{b}$ is consistent if the augmented matrix $[A \ \vec{b}]$ has a pivot position in every row.
(f) True or false? A homogeneous linear system is always consistent.

(g) True or false? The columns of any $4 \times 5$ matrix are linearly dependent.

(h) True or false? Let $A$ be an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ be a nonzero vector. If the solution set of $A\vec{x} = \vec{0}$ is infinite, then the solution set of $A\vec{x} = \vec{b}$ is also infinite.

(i) True or false? The transform $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} x + y \\ y - x \\ 3x \end{bmatrix}$ is linear.

(j) True or false? If $A$ is an $m \times n$ matrix, then the range of the transformation $x \mapsto Ax$ is $\mathbb{R}^m$. 