MATH 221 Summer 2017 Assignment 2

§2.1–§2.3, §2.5 (we skip §2.4)

1. (§2.1#9) Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix}$. What value(s) of $k$, if any, will make $AB = BA$?

2. (§2.1#10) Let $A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 3 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & -5 \\ 2 & 1 \end{bmatrix}$. Verify that $AB = AC$ and yet $B \neq C$.

3. (§2.1#15) The following questions concern arbitrary matrices $A$, $B$, and $C$ for which the indicated sums and products are defined. Mark each statement True or False. Justify each answer.

(a) If $A$ and $B$ are $2 \times 2$ matrices with columns $\vec{a}_1$, $\vec{a}_2$ and $\vec{b}_1$, $\vec{b}_2$, respectively, then $AB = [\vec{a}_1 \vec{b}_1 \quad \vec{a}_2 \vec{b}_2]$.

(b) Each column of $AB$ is a linear combination of the columns of $B$ using weights from the corresponding column of $A$.

(c) $AB + AC = A(B + C)$

(d) $A^T + B^T = (A + B)^T$

(e) The transpose of a product of matrices equals the product of their transposes in the same order.

4. (§2.1#21) Suppose the last column of $AB$ is entirely zeros but $B$ itself has no column of zeros. What can be said about the columns of $A$?

5. (§2.1#22) Show that if the columns of $B$ are linearly dependent, then so are the columns of $AB$.

6. (§2.2#2) Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 \\ 8 & 5 \end{bmatrix}$, if it exists.

7. (§2.2#9) Mark each statement True or False. Justify each answer.

(a) In order for a matrix $B$ to be the inverse of $A$, the equations $AB = I$ and $BA = I$ must both be true.

(b) If $A$ and $B$ are $n \times n$ and invertible, then $A^{-1}B^{-1}$ is the inverse of $AB$.

(c) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ab - cd \neq 0$, then $A$ is invertible.

(d) If $A$ is an invertible $n \times n$ matrix, then the equation $A\vec{x} = \vec{b}$ is consistent for each $\vec{b}$ in $\mathbb{R}^n$.

(e) (skip)

8. (§2.2#31) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$, if it exists.
9. (§2.2#37) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix}$. Construct a $2 \times 3$ matrix $C$ (by trial and error) using only 1, $-1$, and 0 as entries, such that $CA = I_2$. Compute $AC$ and note that $AC \neq I_3$.

10. (§2.3#11) In the following questions, all matrices are $n \times n$. Mark each statement True or False. Justify each answer.

(a) If the equation $A\vec{x} = \vec{0}$ has only the trivial solution, then $A$ is row equivalent to the $n \times n$ identity matrix.

(b) If the columns of $A$ span $\mathbb{R}^n$, then the columns are linearly independent.

(c) If $A$ is an $n \times n$ matrix, then the equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbb{R}^n$.

(d) If the equation $A\vec{x} = \vec{0}$ has a nontrivial solution, then $A$ has fewer than $n$ pivot positions.

(e) If $A^T$ is not invertible, then $A$ is not invertible.

11. (§2.3#17) Can a square matrix with two identical columns be invertible? Why or why not?

12. (§2.3#26) Explain why the columns of $A^2$ span $\mathbb{R}^n$ whenever the columns of an $n \times n$ matrix $A$ are linearly independent.

13. (§2.5#7,9) Let $A = [\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3]$ and

$$\vec{v}_1 = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -3 \\ 8 \\ -7 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -4 \\ 6 \\ -7 \end{bmatrix}, \quad \vec{p} = \begin{bmatrix} -6 \\ -10 \\ 11 \end{bmatrix}.$$

(a) How many vectors are in $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?

(b) How many vectors are in $\text{Col } A$?

(c) Is $\vec{p}$ in $\text{Col } A$? Why or why not?

(d) Is $\vec{p}$ in $\text{Nul } A$? Why or why not?

14. (§2.5#18) Determine if the set of vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 5 \\ 1 \\ -4 \end{bmatrix},$$

is a basis for $\mathbb{R}^3$. Justify the answer.

15. (§2.5#24) The matrix $A = \begin{bmatrix} 3 & -6 & 9 & 0 \\ 2 & -4 & 7 & 2 \\ 3 & -6 & 6 & -6 \end{bmatrix}$ has an echelon form $\begin{bmatrix} 1 & -2 & 5 & 4 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Find a basis for $\text{Col } A$ and a basis for $\text{Nul } A$.

16. (§2.5#36) What can be said about $\text{Nul } C$ when $C$ is a $6 \times 4$ matrix with linearly independent columns?