1. (§1.1#12) Solve the system

\[
\begin{align*}
    x_1 - 5x_2 + 4x_3 &= -3 \\
    2x_1 - 7x_2 + 3x_3 &= -2 \\
    -2x_1 + x_2 + 7x_3 &= -1
\end{align*}
\]

2. (§1.1#16) Determine if the system is consistent. Do not completely solve the system.

\[
\begin{align*}
    2x_1 - 4x_4 &= -10 \\
    3x_2 + 3x_3 &= 0 \\
    x_3 + 4x_4 &= -1 \\
    -3x_1 + 2x_2 + 3x_3 + x_4 &= 5
\end{align*}
\]

3. (§1.1#20) Determine the value(s) of \( h \) such that the matrix is the augmented matrix of a consistent linear system.

\[
\begin{bmatrix}
    1 & h & -5 \\
    1 & -8 & 6
\end{bmatrix}
\]

4. (§1.2#4) Row reduce the matrix to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

\[
\begin{bmatrix}
    1 & 2 & 4 & 5 \\
    2 & 4 & 5 & 4 \\
    4 & 5 & 4 & 2
\end{bmatrix}
\]

5. (§1.2#8) Find the general solutions of the system whose augmented matrix is

\[
\begin{bmatrix}
    1 & -3 & 0 & -5 \\
    -3 & 7 & 0 & 9
\end{bmatrix}
\]

6. (§1.2#11) Find the general solutions of the system whose augmented matrix is

\[
\begin{bmatrix}
    3 & -2 & 4 & 0 \\
    9 & -6 & 12 & 0 \\
    6 & -4 & 8 & 0
\end{bmatrix}
\]

7. (§1.2#24) Suppose a system of linear equations has a \( 3 \times 5 \) augmented matrix whose fifth column is not a pivot column. Is the system consistent? Why (or why not)?

8. (§1.2#26) Suppose a \( 3 \times 5 \) coefficient matrix for a linear system has three pivot columns. Is the system consistent? Why or why not?

9. (§1.3#11) Determine if \( \mathbf{b} \) is a linear combination of \( \mathbf{a}_1, \mathbf{a}_2 \) and \( \mathbf{a}_3 \).

\[
\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}.
\]
10. (§1.3#13) Determine if \( \mathbf{b} \) is a linear combination of the vectors formed from the columns of the matrix \( \mathbf{A} \).

\[
\mathbf{A} = \begin{bmatrix}
1 & -4 & 2 \\
0 & 3 & 5 \\
-2 & 8 & -4
\end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix}
3 \\
-7 \\
-3
\end{bmatrix}.
\]

11. (§1.3#24) Mark each statement True or False. Justify each answer.

(a) When \( \mathbf{u} \) and \( \mathbf{v} \) are nonzero vectors, Span\{\( \mathbf{u}, \mathbf{v} \)\} contains only the line through \( \mathbf{u} \) and the origin, and the line through \( \mathbf{v} \) and the origin.

(b) Any list of five real numbers is a vector in \( \mathbb{R}^5 \).

(c) Asking whether the linear system corresponding to an augmented matrix

\[
[a_1 \ a_2 \ a_3 \ \mathbf{b}]
\]

has a solution amounts to asking whether \( \mathbf{b} \) is in \( \text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\} \).

(d) The vector \( \mathbf{v} \) results when a vector \( \mathbf{u} - \mathbf{v} \) is added to the vector \( \mathbf{v} \).

(e) The weights \( c_1, \ldots, c_p \) in a linear combination \( c_1 \mathbf{v} + \cdots + c_p \mathbf{v} \) cannot all be zero.

12. (§1.4#10) Write the system first as a vector equation and then as a matrix equation.

\[
\begin{align*}
4x_1 - x_2 &= 8 \\
5x_1 + 3x_2 &= 2 \\
3x_1 - x_2 &= 1
\end{align*}
\]

13. (§1.4#17) How many rows of \( \mathbf{A} \) contain a pivot position? Does the equation \( \mathbf{A}\mathbf{x} = \mathbf{b} \) have a solution for each \( \mathbf{b} \in \mathbb{R}^4 \)?

\[
\mathbf{A} = \begin{bmatrix}
1 & 3 & 0 & 3 \\
-1 & -1 & -1 & 1 \\
0 & -4 & 2 & -8 \\
2 & 0 & 3 & -1
\end{bmatrix}
\]

14. (§1.4#19) Can each vector in \( \mathbb{R}^4 \) be written as a linear combination of the columns of the matrix \( \mathbf{A} \) above? Do the columns of the matrix \( \mathbf{A} \) span \( \mathbb{R}^4 \)?

15. (§1.4#24) Mark each statement True of False. Justify each answer.

(a) Every matrix equation \( \mathbf{A}\mathbf{x} = \mathbf{b} \) corresponds to a vector equation with the same solution set.

(b) If the equation \( \mathbf{A}\mathbf{x} = \mathbf{b} \) is consistent, then \( \mathbf{b} \) is in the set spanned by the columns of \( \mathbf{A} \).

(c) Any linear combination of vectors can always be written in the form \( \mathbf{A}\mathbf{x} \) for a suitable matrix \( \mathbf{A} \) and vector \( \mathbf{x} \).

(d) If the coefficient matrix \( \mathbf{A} \) has a pivot position in every row, then the equation \( \mathbf{A}\mathbf{x} = \mathbf{b} \) is inconsistent.

(e) The solution set of a linear system whose augmented matrix is \( [\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3 \ \mathbf{b}] \) is the same as the solution set of \( \mathbf{A}\mathbf{x} = \mathbf{b} \), if \( \mathbf{A} = [\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3] \).

(f) If \( \mathbf{A} \) is an \( m \times n \) matrix whose columns do not span \( \mathbb{R}^m \), then the equation \( \mathbf{A}\mathbf{x} = \mathbf{b} \) is consistent for every \( \mathbf{b} \in \mathbb{R}^m \).