1. Find the equation of the plane that passes through the point (0, 0, 1) with a normal vector \( \langle 1, 1, 1 \rangle \). Furthermore, find if the origin lies on the plane.

\textit{Solution.} The equation is given by

\[ 1 \cdot (x - 0) + 1 \cdot (y - 0) + 1 \cdot (z - 1) = 0, \]

that is,

\[ x + y + z - 1 = 0. \]

The origin does not lie on the plane since \((x, y, z) = (0, 0, 0)\) is not a solution to the above equation.

2. State whether the planes \( x + y + z = 1 \) and \( x - y - z = 0 \) are parallel, intersect, or identical. If they intersect, state whether they are orthogonal.

\textit{Solution.} Two normal vectors for the planes are given by \( \vec{n}_1 = \langle 1, 1, 1 \rangle \) and \( \vec{n}_2 = \langle 1, -1, -1 \rangle \), respectively. Since \( \vec{n}_1 \parallel \vec{n}_2 \), the planes intersect. Since \( \vec{n}_1 \cdot \vec{n}_2 = -1 \neq 0 \), they are not orthogonal.

3. Let \( f(x, y) = \frac{x}{x^2 + y^2} \).

(a) Find the natural domain of \( f \).

(b) Find \( f_x \) and \( f_y \).

(c) A function \( f(x, y) \) is called harmonic if \( f_{xx} + f_{yy} = 0 \) everywhere it is defined. Check if this function \( f \) is harmonic.

\textit{Solution.} (a) Since \( x^2 + y^2 \) is on the denominator, we must have \( x^2 + y^2 \neq 0 \), i.e. \((x, y) \neq (0, 0)\), so the natural domain is the whole plane \( \mathbb{R}^2 \) with the origin removed.

(b) By the quotient rule and the power rule,

\[ f_x = \frac{y^2 - x^2}{(x^2 + y^2)^2}, \quad f_y = -\frac{2xy}{(x^2 + y^2)^2}. \]
(c) By the quotient rule and the power rule again,

\[ f_{xx} = \frac{-2x(x^2 + y^2)^2 - (y^2 - x^2)2(x^2 + y^2)(2x)}{(x^2 + y^2)^4} \]
\[ = \frac{-2x(x^2 + y^2) - (y^2 - x^2)2(2x)}{(x^2 + y^2)^3} \]
\[ = \frac{(-2x)(x^2 + y^2 + 2(y^2 - x^2))}{(x^2 + y^2)^3} \]
\[ = \frac{-2x(3y^2 - x^2)}{(x^2 + y^2)^3}. \]

Similarly,

\[ f_{yy} = (-2x) \left( \frac{y}{(x^2 + y^2)^2} \right)_{yy} \]
\[ = (-2x) \left( \frac{y}{(x^2 + y^2)^2} \right)_{yy} \]
\[ = (-2x) \frac{x^2 + y^2 - 2y(2y)}{(x^2 + y^2)^4} \]
\[ = (-2x) \frac{x^2 - 3y^2}{(x^2 + y^2)^3}. \]

Hence \( f_{xx} + f_{yy} = 0 \), so \( f \) is harmonic. 

\[ \square \]