## A Quick Revision

<table>
<thead>
<tr>
<th>Task</th>
<th>Local</th>
<th>Global</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Steps</strong></td>
<td><strong>Analyse critical points</strong></td>
<td><strong>Find absolute max/min over a region.</strong></td>
</tr>
</tbody>
</table>

1. Find the critical points by solving
   \[ \nabla f = 0, \text{ i.e.} \]
   \[ \left\{ \begin{array}{l}
   f_x(a,b) = 0 \\
   f_y(a,b) = 0
   \end{array} \right. \]

2. Find \( f_{xx}, f_{xy} \), and compute
   \[ D(x,y) = \det(Hf(x,y)) \]
   \[ = f_{xx}f_{yy} - f_{xy}^2 \]

3. \[ \left\{ \begin{array}{l}
   D > 0, \ f_{xx} > 0 \Rightarrow \text{loc. min} \\
   D > 0, \ f_{xx} < 0 \Rightarrow \text{loc. max} \\
   D < 0 \Rightarrow \text{saddle pt.} \\
   D = 0 \Rightarrow \text{inconclusive}
   \end{array} \right. \]

1. The same as \( \leq \), + recording values at critical points, in a table.
2. Find max/min on the boundary.

**Option 1:** Eliminating \( x \) or \( y \) \( \Rightarrow \) Optimisation w.r.t. to a single variable.

**Option 2:** Lagrange multiplier method.

Record the max/min on the boundary.
3. Compare the recorded values at critical points and on the boundary.
Comparison between options to find max/min on the boundary.

<table>
<thead>
<tr>
<th>Elimination of $x$ or $y$</th>
<th>Lagrange multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundaries that cannot be expressed as given by a single equation, e.g. rectangular/triangular boundary</td>
<td>✓</td>
</tr>
<tr>
<td>Analyse each part of the boundary separately</td>
<td></td>
</tr>
<tr>
<td>Boundaries expressible by a single equation, e.g. circular/elliptical/elliptical/unfamiliar equation like $x^2 + xy + y^2 = 2$</td>
<td>×</td>
</tr>
<tr>
<td>It could be done, but considerably hard.</td>
<td></td>
</tr>
</tbody>
</table>
Q1: A car moves at a uniform speed of 50 km/h, from 12 pm to 4 pm. How far has the car travelled?

Solution: Distance = 50 \times 4 = 200 \text{ km}

Area of rectangle

\begin{align*}
&= 50 \times 4 = 200 \\
\Rightarrow & \text{Area of rectangle} \\
&= \text{Distance the car has travelled}.
\end{align*}

Q2: A car starts with constant acceleration 0.5 m/s\(^2\). How far has the car travelled from t = 0 to t = 10 s?
Sol: Acceleration = the slope of the line that describes the relation between velocity and time.

The distance the car has travelled

= Area of the triangle
= Area between the graph of \( v = 5t \), the straight lines \( t=10 \) and \( t=0 \).

\[
\frac{1}{2} \times 10 \times (5 \times 10) = 250 \text{ m}.
\]

Q3. A car moves on a straight road with velocity described by the equation \( v = t^2 \) from 0 to 10 s. How far has the car travelled?
Sol: By our observation before,

distance = the area between the straight lines

t = 0, t = 10

and the graph of \( v = t^2 \).

Historically, this was exactly why Newton and Leibniz invented calculus.
(Actually, integration preceded differentiation!)

Attempts to find the area above:

Approximating:
"pretend" the car is move at 10m/s constant rate.

Rectangular area

\[ = 100 \times 10 = 1000. \]

Triangular area

\[ = \frac{1}{2} \times 10 \times 100 = 500. \]

Rectangular area

\[ = 5 \times 5^2 + 5 \times 10^2 = 625. \]

Triangular area (Trapezoidal)

\[ = 375. \]
Rectangular area

\[ = 2 \times 2^2 + 2 \times 4^2 + 2 \times 6^2 + 2 \times 8^2 + 2 \times 10^2 \]

\[ = 440 \]

Trapezoidal area = 340